Managing Uncertainty in a Supply Chain: Safety Inventory

What is safety inventory?
- The average inventory remaining when the replenishment lot arrives

Why need safety inventory?
- To manage
  - demand uncertainty
  - Supply uncertainty

Raising the level of safety inventory
- increases product availability
- increases inventory holding costs

(A) With variable demand
- The average inventory would be 100 units when demand (20/d) and lead time (10/day) were constant
- If demand was actually 25/d, inventory would be depleted by 8 (200/25). And there would be a stockout of 50 units.
- A safety stock of 50 units would prevent a stockout due to variation in demand

(B) With variable lead time
- The lead time can vary by plus or minus two days
- When the order arrived 2 days late, on day 12, the firm would experience stockouts for 2 days (40 units)
- If the management believed that shipments would never arrive more than two days late, a safety stock of 40 units would ensure a stockout would not occur. This would require holding an average inventory of 140 units
With variable demand and lead time

- The demand is above forecast by the maximum, 25 units and the incoming order arrives two days late.
- The result is a stockout period of four days at 25 units per day.
- If management wanted to protect against the maximum variability in both demand and lead time, the firm would need a safety stock of 100 units. This policy (no stockout) would result in an average inventory of 200 units.

Safety Inventory

- **Two key questions**
  - What is the appropriate level of safety inventory to carry?
  - What actions can be taken to improve product availability while reducing safety inventory?

- **A key to success**
  - To decrease the level of safety inventory carried without hurting the level of product availability
  - Successful stories: Dell (compared with Compaq), Wal-Mart, 7-11 Japan

- **Factors influencing safety stocks**
  - Forecast error (demand uncertainty)
  - Lead time (supply uncertainty)
  - Exposure to stockout
  - Service level required (product availability)

Measuring uncertainties and product availability

- **Measures of demand uncertainty**
  - Average demand per period, $D$
  - Standard deviation of demand per period, $\sigma_D$
  - Coefficient of variation, $cv$
    - $cv = \frac{\sigma_D}{\mu}$

- **Measures of product availability**
  - Product fill rate, $fr$
  - The fraction of product demand that is satisfied from production in inventory
  - Order fill rate
    - The fraction of orders that are filled from available inventory
  - Cycle service level, $CSL$
    - The fraction of replenishment cycles (the interval between two successive replenishment deliveries) that end with all the customer demand being met
    - Equals to the probability of not having a stockout in a replenishment cycle

2.3 Basic Inventory Replenishment Policies

1) **Continuous review systems** – ($Q$, ROP)
   - Order fixed quantity ($Q$) when total inventory drops below Reorder Point (ROP)
   - Fixed order size and varied order interval

2) **Periodic review systems** – ($T$, OUL)
   - Order at fixed time intervals ($T$) to raise total inventory to Order up to Level (OUL)
   - Varied order size and fixed order interval

3) **The combination of 1) and 2)** – (ROP, OUL)
   - Fixed reorder point and fixed maximum inventory level
Continuous Review Policy:

Periodic Review Policy:

Evaluation of Replenishment Policies

- Given a replenishment policy, determine
  - Safety inventory (see Example 11.1)
  - Cycle service level (see Example 11.2)
  - Fill rate (see Example 11.3)

- Given desired product availability, design replenishment policy
  - Given desired Cycle Service level, determine safety inventory (see Example 11.4)
  - Given desired Fill Rate, determine required safety inventory (see Example 11.5)

3. Continuous Review Policy:

Determining Safety Inventory and Cycle Service Level

\[
L: \text{Lead time for replenishment} \\
D: \text{Average demand per unit time} \\
\sigma_D: \text{Standard deviation of demand per period} \\
R: \text{Mean demand during lead time} \\
\sigma_L: \text{Standard deviation of demand during lead time} \\
CSL: \text{Cycle service level} = \text{Prob}(\text{demand during lead time} = \text{ROP}) \\
ss: \text{Safety inventory} \\
ROP: \text{Reorder point} \\
\]

\[
D_L = DL \\
\sigma_L = \sqrt{L} \sigma_D \\
ss = F^{-1}(CSL) \times \sigma_L \\
ROP = D_L + ss \\
CSL = F(ROP, D_L, \sigma_L) \\
\]

Average Inventory = \( Q/2 + ss \)

Example 11.1-2 - Determine safety inventory and cycle service level of a given policy

Given the replenishment policy

- \( D = 2500 \) /week
- \( \sigma_D = 500 \)

Average lead time: \( L = 2 \) weeks;

Given continuous review policy: \( (Q, POR) = (10000, 6000) \)

\[
ss = \text{Average Inventory} = \text{cycle inventory} + ss \\
\text{Average Flow Time} = \text{The demand during lead time, } D_L = \text{The standard deviation of the demand in lead time, } \sigma_L = \text{Cycle service level, } CSL = F(ROP, D_L, \sigma_L) = \text{NORMDIST}(ROP, D_L, \sigma_L, 1) \]

ROP = D_L + ss
Continuous Review Policy:
Evaluating Fill Rate

- $f_r$: Proportion of customer demand satisfied from stock
- $ESC$: the expected shortage per cycle

\[
fr = 1 - \frac{ESC}{Q}
\]

\[
ESC = -ss\left[1 - F_S\left(\frac{SS}{\sigma_L}\right)\right] + \sigma_L F_S\left(\frac{SS}{\sigma_L}\right)
\]

In Excel:

$$ESC = -ss\{1-NORMDIST(ss/\sigma_L, 0, 1, 1)} + \sigma_L NORMDIST(ss/\sigma_L, 0, 1, 0)$$

Example 11.3: - Determine Fill Rate of a given policy

Following Example 11.1-2, given $DL=5,000$, $\sigma_L=707$, and $(Q, ROP) = (1000, 6000)$

\[
ss = ROP-DL = 1000
\]

\[
ESC = -1,000\{1-NORMDIST(1,000/707, 0, 1, 1)} + 707 NORMDIST(1,000/707, 0, 1, 0)
\]

\[
fr = (Q - ESC)/Q = (10,000 - 25.13)/10,000 = 0.9975.
\]

Refer to the textbook page 305 for Excel computation.

Key Point:

- Both filling rate and cycle service level increase as the safety inventory is increased.
- For the same safety inventory, an increase in lot size increases the fill rate but not the cycle service level.

Example 11.4: - Determine safety inventory and the policy satisfying desired CSL

Given

- $D = 2,500$/week;
- $\sigma_D = 500$
- $L = 2$ weeks;
- $Q = 10,000$;
- $CSL = P(Demand during lead time \leq ROP) = F(R_L+ss, D_L, \sigma_L) = 0.90$

\[
D_L = \frac{\sigma_L}{\sqrt{2L}}
\]

\[
ss = F_S(1-CSL)\sigma_L
\]

\[
ROP = \frac{D_L}{\sqrt{2L}}
\]

The replenishment policy: $(Q, ROP)$:
Example 11.5: Evaluating Required Safety Inventory

If desired fill rate is \( fr = 0.975 \), how much safety inventory should be held?

**Step 1:**
\[
ESC = (1 - fr)Q = 250
\]

**Step 2:** Solve (by using GOALSEEK tool in Excel or trial-and-error method)
\[
250 = -ss \left[ 1 - \text{NORMDIST} \left( \frac{ss}{\sigma_s} \right) \right] + \sigma_L \text{NORMDIST} \left( \frac{ss}{\sigma_L}, 1, 0 \right)
\]

Safety inventory: \( ss = 67 \)

See P308 of the textbook for Excel computation.

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**Key Managerial Levers**

-- to reduce the level of safety inventory required in a way that does not affect product availability

1. Reduce the supplier lead time, \( L \)
   - If lead time decreases by a factor of \( k \), the required safety inventory decreases by a factor of \( \sqrt{k} \)
   - As the reduction of \( L \) requires a significant effort from the supplier whereas the reduction in \( ss \) occurs at the retailer. It is important for retailer to share some of the resulting benefits

2. Reduce underlying uncertainty of demand, \( \sigma_D \)
   - If \( \sigma_D \) is reduced by a factor of \( k \), the required safety inventory also decreases by a factor \( k \).
   - Better market intelligence
   - Better forecasts
   - Better information sharing along the supply chain

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**4. Periodic Review Policy**

- \( L \): Average lead time
- \( T \): Review (order) interval
- \( D \): Average demand per unit time
- \( \sigma_D \): Standard deviation of demand per unit time
- \( \sigma_{LT} \): Standard deviation of demand during \( L + T \) periods
- \( CSL \): Cycle service level
  - \( P(\text{demand during } L + T \leq \text{OUL}) \)
- \( OUL \): Order up to level
- \( ss \): Safety inventory
- \( Q \): Average lot size

\[
D_{LT} = (T + L)D \\
\sigma_{LT} = \sqrt{\sigma_D^2 + L + T} \\
ss = F_{CSL}^{-1}(CSL) \times \sigma_{LT} \\
OUL = D_{LT} + ss \\
Q = D_t = DT
\]
Example 11.10: Periodic Review Policy
(Lego at a Wal-Mart store)

Given
\[ D = 2500 \text{/week}; \]
\[ \sigma_D = 500 \]
\[ L = 2 \text{ weeks}; \]
\[ T = 4 \text{ weeks}; \]
\[ CSL = 0.90 \]

What is the required safety inventory?

Mean demand during \( T+L \) periods,
\[ D_{T+L} = \]

Standard deviation of demand during \( T+L \) periods,
\[ \sigma_{T+L} = \]

The required safety inventory, \( ss = F_{s-1}(CSL) \sigma_{T+L} = \]

The order up to level, \( OUL = \]

The Periodic Review Policy: \( (T, OUL) = \]

Comparison of the Two Policies

- Comparing Example 11.4 and 11.10
  - Given the same CSL=0.90,
    \[ ss = 906 \text{ boxes when using continuous review policy} \]
    \[ ss = 16570 \text{ boxes when using periodic review policy} \]

Key Points
- With a continuous review policy, the safety inventory is used to cover for demand uncertainty over the lead time. With a periodic review policy, the safety inventory is used to cover for demand uncertainty over the lead time and the review interval, \( T+L \), in which higher uncertainty must be accounted
- Periodic review replenishment policies require more safety inventory than continuous review policies for the the same lead time and level of product availability
- Periodic review is simple to implement and does not require continuous tracking of the inventory
- Companies may partition their products based on their value. High-value products are managed using continuous review policies and low-value products managed using periodic review policies

Strategies to reduce safety inventory without affecting product availability

- A reduction of supply uncertainty (Example 11.6)
- Aggregation reduces demand uncertainty and thus the required safety inventory (Example 11.7)
- Information centralization (Wal-Mart case)
- Specialization (Example 11.8)
- Product substitution
- Component commonality (Ex.11.9)
- Postponement (Dell case)

(All using continuous review policies)

Impact of Supply Uncertainty

- Manage the component inventory for the Dell assembly plant at Austin
  - Demand uncertainty: demand during \( L \) is Normally distributed
  - Supply uncertainty: lead time is Normally distributed

\[ D_l = D_L \]
\[ \sigma_L = \sqrt{\sigma_D^2 + D^2 \sigma_L^2} \]
Example 11.6: Impact of Supply Uncertainty on Safety Inventory

Given:
- \( D = 2,500/\text{day}; \) \( \sigma_D = 500, \) \( L = 7 \text{ days}; \) \( s_L = 7 \text{ days}, \)
- \( Q = 10,000; \) CSL = 0.90

\[ D_t = DL \]
\[ \sigma_t = \]
\[ ss = F_t^{-1}(\text{CSL}) \]

When \( s_L = 0, \)
- \( ss(\text{units}) = 1,695 \) \( ss(\text{days}) = 0.68 \)
- \( = 1, \) \( = 3,625 \) \( = 1.45 \)
- \( = 2, \) \( = 6,628 \) \( = 2.65 \)
- \( = 3, \) \( = 9,760 \) \( = 3.90 \)
- \( = 4, \) \( = 12,927 \) \( = 5.17 \)
- \( = 5, \) \( = 16,109 \) \( = 6.14 \)
- \( = 6, \) \( = 19,298 \) \( = 7.72 \)

Key point:
- A reduction in supply uncertainty can help reduce safety inventory required without hurting product availability

Impact of Inventory Aggregation on Safety Inventory

- **Key questions**
  - How aggregation affects safety inventory?
  - How supply chain can exploit inventory aggregation to reduce the level of safety inventory required without hurting product availability?

Accurate Response: The Impact of Inventory Pooling

Which of the two systems provides a higher level of service for a given level of safety stock?

System A (Decentralized)

\[ (D, \sigma^A) \]

System B (Centralized)

\[ (D^C, \sigma^C) \]

Aggregate Demand:

\[ D^C = \sum_{i=1}^n D_i; \quad \sigma^C = \sqrt{\text{Var}(D^C)} = \sqrt{\sum_{i=1}^n \sigma_i^2 + 2 \sum_{i<j} \text{cov}(i,j)} \]
When demand across all regions is independent
\[ \text{cov}(i,j) = \rho_{ij} \sigma_i \sigma_j = 0, \text{var}(D) = \sum \sigma_i^2, \sigma_D^2 = \sqrt{\text{var}(D)} = \sqrt{\sum \sigma_i^2} \]

\[ \sigma_D^2 \leq \sum \sigma_i^2 \]
When the demands being aggregated are independent, the s.d. of aggregated demand is less than the sum of the s.d. of individual demands

When individual demand is perfectly positively correlated
\[ \rho_{ij} = 1, \text{var}(D) = (\sigma_D^2) = \sum \sigma_i^2 + 2 \sum_{ij} \text{cov}(i,j) = \sum \sigma_i^2 + 2 \sum_{ij} \sigma_i \sigma_j = (\sum \sigma_i)^2 \]

\[ \sigma_D^2 = \sum \sigma_i^2 \]
When the demands being aggregated are perfectly positively correlated, the s.d. of aggregated demand is the sum of the s.d. of individual demands

Example 11.7 - A luxury car dealership at Chicago has 4 retail outlets

Given at each outlet,
- Weekly demand mean, \( D = 25 \) cars
- Standard deviation, \( \sigma_D = 5 \)
- \( L = 2 \) weeks
- Targeted CSL = 0.9

Disaggregated option: (assuming \( \rho_{ij} = 0 \))
\[ \sigma_i = \sqrt{2 \sigma_D^2} = 1.4142 \times 5 = 7.07 \]
\[ \text{ss} = F_{\text{ss}}^{-1}(\text{CSL}) \]

\[ \sigma_{\text{ss}} = \frac{\text{ss}}{NORMSINV(0.9) \times 7.07} = 9.06 \text{ (cars/per outlet)} \]

Total ss required = \( 4 \times 9.06 = 36.24 \) cars

Aggregated option: (with a central outlet)
\[ \sigma_{DC} = 4 \times 25 = 100 \]
\[ \sigma_{\text{ss}} = \frac{\text{ss}}{NORMSINV(0.9) \times 14.14} = 18.12 \]

Impact of Correlation on Benefit From Aggregation

Key points
- Aggregation reduces demand uncertainty and thus the required safety inventory as long as the demand being aggregated is not perfectly positively correlated (Examples: Dell, Gateway ...
- The square root law:
  - If the number of independent stocking locations decreases by a factor of \( n \), the average inventory is expected to decrease by a factor of \( n \)
- In general, demand being aggregated is unlikely to be perfectly independent
- The physical aggregation of inventories in one location may not be optimal due to (Examples: Gap, McMaster-Carr, Amazon.com ...)
  - Increased response time to customer order
  - Increased transportation cost to customer
- There are clear benefits to aggregating safety inventory
Methods to Extract Benefits of Aggregation

- Physical Centralization
- Information Centralization
- Specialization
- Product substitution
- Component commonality
- postponement

Williams Sonoma and Laura Ashley as examples of Physical centralization. Mail order companies with multiple warehouses as examples of Information centralization. Also Wal-Mart and The Gap allow for Information Centralization cutting the level of safety stock carried. Bennetton for raw material commonality. The major benefit here is that since greige goods are to be ordered, the estimate is much more accurate since it aggregates across all colors. Also refer to the HP Laser Jet example.

Information Centralization

- McMaster-Carr: – Virtual aggregating inventory
- Gap: – Virtually aggregate inventory across all retail stores, even though the inventory is physically separated
- Wal-Mart: – uses information centralization with a responsive transportation system (to allow store managers to exchange products) to reduce the amount of safety inventory carried while providing a high level of product availability

Component Commonality

- A very effective supply chain strategy to exploit aggregation and reduce component inventories
- Component commonality decreases the safety inventory required. The marginal benefit, however, decreases with increasing commonality

Postponement

- The supply chain ability in organizing the production and distribution of products in such a way that the customization of these products is made as close to the point when the demand is known as possible
- A reactive lever for reduction of the demand and supply variability
- The ultimate goal is to improve the flexibility capabilities of the firm, and thus to improve profitability by better matching supply with demand
- A powerful concept for e-commerce channel
- Successful examples: HP, Motorola,...
Postponement is the ability of a supply chain to delay product customization or differentiation until closer to the time the product is sold.

Example: The PC company
- A PC company that manufactures and sells netebook PCs directly to users
- One day ship is guaranteed
- A new model has 25 variations (software and RAM...)
- Same demand for each of 25 variation: a normal distribution, with an average of 100/day and a s.d. of 100

1. Without postponement:
   \[ \mu = 100/day, \sigma = 100, \text{ service level } = 95\% \]
   Total inventory needed at beginning of the day = \[ 25(\mu + 1.65\sigma) = 25(100+1.65\times100) = 6625 \text{ units} \]

2. Postpone installing the software and RAM until after the order has been received while maintaining 95% of the PCs be shipped that day
   \[ \mu = 100/day, \sigma = 100, \text{ service level } = 95\% \]
   Total inventory needed at beginning of the day = \[ 25 \times \mu + 1.65 \sqrt{25\sigma} = 25(100+1.65\times5\times100) = 3325 \text{ units} \]

The inventory level is cut virtually in half!! If each PC costs $3000, then the reduction in inventory is $9.9 million.
Estimating and Managing Safety Inventory in Practice

- Account for the fact that supply chain demand is lumpy
- Adjust inventory policies if demand is seasonal
- Use simulation to test inventory policies
- Start with a pilot
- Monitor service level
- Focus on reducing safety inventories...

The Supply Chain of HP's Ink-jet Printer:

- Suppliers
- IC Mfg
- PCAT
- Print Mech Mfg
- Factories
- distributors
- DC's

Key:
- IC Mfg: Integrated circuit manufacturing
- PCAT: Printed circuit assembly and test
- Print Mech Mfg: Print mechanism manufacturing

Concepts:
- Component commonality
- Centralization
- Aggregation
- Postponement

Customization Costs Drop with the Generic DeskJet Printer

- Manufacturing costs
- Freight and duty costs
- Factory inventory costs
- Distribution center and transit inventory costs

Annual cost in millions of dollars

Factory
Distribution center
Where customization occurs