ON THE SAMPLING PERFORMANCE OF AN IMPROVED STEIN INEQUALITY RESTRICTED ESTIMATOR

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Summary

Using the Stein (1964) variance estimator, this paper defines a modified Stein inequality constrained estimator and derives its exact risk under quadratic loss. Numerical evaluations show that over a wide range of the parameter space, the modified Stein inequality constrained estimator has lower risk than the traditional Stein inequality constrained estimator introduced by Judge et al. (1984).

Key words: inequality constraints; quadratic loss; Stein-rule.

1. Introduction

In recent years, an extensive literature has developed on Stein-rule estimation of econometric models (see e.g. Judge & Bock, 1978). It is well documented and understood that in regression, the Stein-rule estimator leads to uniform improvement over the ordinary least squares (OLS) estimator in terms of risk under quadratic loss, provided that there are at least three coefficients. It is also well known that in estimating the normal variance, the Stein (1964) variance estimator dominates the usual variance estimator. Gelfand & Dey (1988) and Ohtani (1988) showed that in the context of regression, the Stein (1964) estimator may be viewed as a pre-test estimator after the pre-test for a linear hypothesis on the coefficients. More recently, Berry (1994) and Ohtani (1996) respectively showed that the Stein-rule estimators of the mean vector of a multivariate normal population, and the coefficient vector of a linear regression model, can both be improved by incorporating the Stein (1964) variance estimator.

In regression analysis, inequality constraints frequently arise on the unknown parameters. For example, it is a common practice to delete a variable if its coefficient turns out to have an algebraic sign contrary to a priori expectation. Within the context of the linear regression model, there is a considerable body of literature relating to the sampling properties of the resulting inequality constrained least squares (ICLS) estimator. Examples are Judge & Yancey (1981, 1986), Thomson & Schmidt (1982), Ohtani (1991) and Wan (1994a, b). For the orthonormal linear model and a quadratic loss measure, Judge et al. (1984) used the Stein-rule estimator to specify a Stein inequality constrained (SIC) estimator which demonstrates the inadmissibility of the ICLS estimator and thus demonstrates the latter estimator’s inadmissibility.

In this paper, building on the work of Berry (1994) and Ohtani (1996), we consider a modified Stein inequality constrained (MSIC) estimator which incorporates the Stein (1964) variance estimator.
variance estimator. We show that the MSIC estimator is a pre-test estimator after two preliminary tests, and under quadratic loss the risks associated with the MSIC estimator are lower than those of the ICLS and SIC estimators for a wide range of parameter values.

2. Notations and Estimators

We are concerned with estimating the regression coefficients in the model

\[ Y = X\beta + \epsilon \quad (\epsilon \sim N(0, \sigma^2 I)), \]  

where \( Y \) and \( \epsilon \) are \( n \times 1 \) vectors, \( \beta \) is \( k \times 1 \) and \( X \) is \( n \times k \), non-stochastic and of rank \( k \). In addition to sample information, we assume there exists prior information that a linear combination of the coefficients satisfies the inequality constraint

\[ C'\beta \geq 0, \]  

where \( C \) is a \( 1 \times k \) vector. For convenience, we follow Judge & Yancey (1981, 1986) and reparameterize (2.1) and (2.2) as

\[ Y = Z\theta + \epsilon \quad \text{and} \quad \theta_1 \geq 0, \]  

respectively, where \( Z = XS^{-1/2}Q' \), \( S = X'X \), \( \theta = QS^{1/2}\beta \), \( \theta_1 \) is the first element of \( \theta \) and \( Q \) is an orthogonal matrix such that

\[ QS^{-1/2}C(C'S^{-1}C)^{-1}C'S^{-1/2}Q' = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \]

The ICLS estimator of \( \theta \) is given by

\[ \hat{\theta} = I[\hat{\theta}_1 \geq 0]\hat{\theta} + I[\hat{\theta}_1 < 0]I(A), \]  

where \( \hat{\theta} = Z'Y \) is the OLS estimator of \( \theta \), \( I(\theta^*) = (0, \hat{\theta}_1^*, \hat{\theta}_2^*)' \) is the equality constrained estimator of \( \theta^* \), \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \) are respectively the first and remaining \( k - 1 \) elements of \( \hat{\theta} \), and \( I(A) \) is an indicator function which is 1 if event \( A \) occurs and 0 otherwise. Judge & Yancey (1981, 1986) and Thomson & Schmidt (1982) found that \( \hat{\theta} \) is biased, but under the quadratic loss measure \( \hat{\theta} \) is non-trivially superior to \( \theta^* \) over a wide range of the parameter space. Given that \( \hat{\theta} \) is dominated by the Stein-rule estimator when \( k \geq 3 \), a question arises as to the sampling performance of the estimator that results when \( \hat{\theta} \) and \( \theta^* \) are replaced by their corresponding Stein-rule estimators in (2.3). This question motivated Judge et al. (1984) to consider the following SIC estimator,

\[ \hat{\theta}_S = I[\hat{\theta}_1 \geq 0]\hat{\theta} + I[\hat{\theta}_1 < 0]I(A). \]

where \( 0 \leq \eta_1 \leq 2(k - 2) \) and \( 0 \leq \eta_2 \leq 2(k - 3) \). Assuming that \( \sigma^2 \) is known, Judge et al. (1984) showed that for \( k \geq 4 \), \( \hat{\theta}_S \) strictly dominates \( \hat{\theta} \). Wan (1994c), among other things, generalized Judge’s results to include an interval constraint.
Following suggestions made by George (1990), Berry (1994) showed that the Stein-rule estimator for estimating the multivariate normal mean vector can be improved by incorporating the Stein (1964) variance estimator in estimating $\sigma^2$. In the present context, the Stein unconstrained variance estimator may be written as

$$\hat{\sigma}_s^2 = \min \left( \frac{e'e}{v+2}, \frac{Y'Y}{n+2} \right),$$

where $v = n - k$. Note that $Y'Y = e'e + \tilde{\theta}'\tilde{\theta}$, and that $e'e/(v+2) \leq Y'Y/(n+2)$ if and only if $F_1 \geq v/(v+2)$, where $F_1 = (\tilde{\theta}'\tilde{\theta}/k)/(e'e/v)$ is the test statistic for testing the hypothesis $H_0$: $\theta = 0$ versus $H_1$: $\theta \neq 0$. Thus the Stein unconstrained variance estimator can be viewed as the pre-test estimator after the pre-test of $H_0$ against $H_1$. That is,

$$\hat{\sigma}_s^2 = \begin{cases} 
\frac{e'e}{v+2} & \text{if } F_1 \geq c_1, \\
\frac{Y'Y}{n+2} & \text{if } F_1 < c_1,
\end{cases} \quad (2.5)$$

where $c_1 = v/(v+2)$. Using (2.5), we obtain the following modified Stein unconstrained (MSU) estimator of $\theta$,

$$\hat{\theta}_{ms} = I(F_1 \geq c_1) \left( \frac{1}{\frac{a_1e'e}{\tilde{\theta}'\tilde{\theta}}} \hat{\theta} \right) + I(F_1 < c_1) \left( \frac{1}{\frac{a_2Y'Y}{\tilde{\theta}'\tilde{\theta}}} \hat{\theta} \right)$$

$$= I(F_1 \geq c_1) \left( \frac{1}{\frac{a_1e'e}{\tilde{\theta}'\tilde{\theta}}} \hat{\theta} \right) + I(F_1 < c_1) \left[ \left( 1 - a_2 \left( 1 + \frac{e'e}{\tilde{\theta}'\tilde{\theta}} \right) \right) \hat{\theta} \right],$$

where $0 \leq a_1 \leq 2(k-2)/(v+2)$ and $0 \leq a_2 \leq 2(k-2)/(n+2)$. Berry’s (1994) estimator is equivalent to $\hat{\theta}_{ms}$ with $c_1 = k/(v+2)$. Ohtani (1996) shows that the estimator studied by Berry (1994) can be further improved by considering a broader range of critical values for the pre-test.

To motivate the modified Stein equality constrained (MSEC) estimator of $\theta$, we consider testing the hypothesis $H_2$: $\tilde{\theta}_2 = 0$ versus $H_3$: $\tilde{\theta}_2 \neq 0$ based on $\theta^e$, the equality constrained estimator. The test statistic is $F_2 = (\tilde{\theta}'\tilde{\theta}/(k-1))/\left(\tilde{e}'\tilde{e}/(v+1)\right)$, where $\tilde{e} = e + Z(\hat{\theta}_1, 0')$. Under $H_2$, $\sigma^2$ is estimated by $Y'Y/(n+2)$. Under $H_3$, it is estimated by $(e'e + \tilde{\theta}_2^2)/(v+3)$. Note that $(e'e + \tilde{\theta}_2^2)/(v+3) \leq Y'Y/(n+2)$ if and only if $F_2 \geq (v+1)/(v+3)$. Thus, the equality constrained Stein variance estimator may be written as

$$\sigma_{s2}^2 = \begin{cases} 
\frac{e'e + \tilde{\theta}_2^2}{v+3} & \text{if } F_2 \geq c_2, \\
\frac{Y'Y}{n+2} & \text{if } F_2 < c_2,
\end{cases}$$

where $c_2 = (v+1)/(v+3)$. Accordingly, $\theta_{ms}^e$, the MSEC estimator of $\theta$ may be expressed as

$$I(F_2 \geq c_2) \left[ \left( 1 - \frac{a_3(e'e + \tilde{\theta}_2^2)}{\tilde{\theta}_2^2} \right) \hat{\theta}_2 \right] + I(F_2 < c_2) \left[ \left( 1 - a_4 \left[ 1 + \frac{e'e + \tilde{\theta}_2^2}{\tilde{\theta}_2^2} \right) \right) \hat{\theta}_2 \right].$$

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where \(0 \leq a_3 \leq 2(k - 3)/(v + 3)\) and \(0 \leq a_4 \leq 2(k - 3)/(n + 2)\).

If \(\hat{\theta}_{mS}\) and \(\hat{\theta}^*_{mS}\) are substituted for the component estimators in (2.4), we obtain the MSIC estimator

\[
\hat{\theta}_{mS} = I[\hat{\theta}_1 \geq 0]\hat{\theta}_{mS} + I[\hat{\theta}_1 < 0]\hat{\theta}^*_{mS}.
\]

If \(c_1 = c_2 = 0\), then \(\hat{\theta}_{mS}\) collapses to \(\hat{\theta}_S\), the SIC estimator. The properties of \(\hat{\theta}_S\) have only been considered in the case of known \(\sigma^2\) (see Judge et al., 1984). Next, we derive and evaluate the risk of \(\hat{\theta}_{mS}\) under a scaled quadratic loss function.

3. Risk of the Modified Stein Inequality Constrained (MSIC) Estimator

For an estimator \(\hat{\theta}\) of \(\theta\), its risk under quadratic loss scaled by \(\sigma^2\) is defined by

\[
R(\hat{\theta}) = E[(\hat{\theta} - \theta)'(\hat{\theta} - \theta)/\sigma^2].
\]

**Theorem 1.** Under the stated assumptions, the risk of \(\hat{\theta}_{mS}\) is given by \(R(\hat{\theta}_{mS})\), which equals

\[
k + \left(\frac{(\theta_1/\sigma)^2 - 1}{\Phi(-\theta_1/\sigma)} - (\theta_1/\sigma)\psi(-\theta_1/\sigma) + a_1^2 J(0, 0, 0, 0, 0) + J(0, 0, 1, 0, 1, 1, 1) + a_1^2 J(0, 0, 1, 0, 0) + J(0, 0, 1, 0, 1, 1, 1) + a_1^2 (H(0, -1, 2, 1) + H(4, -1, 0, 1) + 2H(2, -1, 1, 1)) + a_1^2 (H(0, 1, 0, 0) + H(0, -1, 2, 0) + H(4, -1, 0, 0) + 2H(2, -1, 1, 1)) - 2a_2 (J(0, 0, 1, 0, 1) - (\theta_1/\sigma) \sigma J(1, 0, 0, 1, 1) - L(0, 0, 0, 1, 1)) - 2a_2 (J(2, 0, 0, 0, 0) - \theta_1/\sigma) \sigma J(1, 0, 0, 0, 0) - L(0, 0, 0, 0, 0) + J(0, 0, 1, 0, 0) - (\theta_1/\sigma) \sigma J(1, 0, 0, 1, 0) - L(0, 0, 0, 1, 0)) - 2a_2 (H(0, 0, 1, 1) + H(2, 0, 0, 1) - G(0, -1, 1, 1) - G(2, -1, 0, 1)) - 2a_2 (H(0, 1, 0, 0) - G(0, 0, 0, 0, 0) + H(0, 1, 0, 0) + H(2, 0, 0, 0) - G(0, -1, 1, 0) - G(2, -1, 0, 0)) + 2a_1^2 J(0, 0, 1, 0, 0) + 2a_1^2 (H(0, 0, 1, 0, 0) + H(2, 0, 0, 0)),
\]

(3.1)

where

\[
\varphi(x) = e^{-x^2/2}/\sqrt{2\pi}, \quad \Phi(x) = \int_{-\infty}^{x} \varphi(z) dz,
\]

\[
G(a, b, c, w) = \sigma^2 \left(\lambda_1 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \xi(-1)^{j+a} h_{i+1, j}(a, b, c, w)\right),
\]

\[
H(a, b, c, w) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \xi(-1)^{j+a} h_{i+1, j}(a, b, c, w),
\]

\[
J(a, b, c, d, w) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \xi g_{ij}(a, b, c, d, w),
\]

\[
L(a, b, c, d, w) = \sigma^2 \left(\lambda_1 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \xi g_{i+1, j}(a, b, c, d, w)\right),
\]

\[
\xi = \sigma^{a+2(b+c)-2} \pi^{-\frac{3}{2}} 2^{-\frac{1}{2} (j+a)+b+c-1} W_j(\theta_1) W_i(\lambda_1),
\]
which is the risk of $O_1$. The higher the value of $RE_0$, $mib$ is much lower than for $mS$.

Illustrates the results for $nk$ smaller than $O_1$ true. The risks of $O_1$ compare to the risk of $mS$ such that they provide the minimal risk for their corresponding component estimators. To analyze the estimator, as shown in Judge & Yancey (1981, 1986) and Thomson & Schmidt (1982), the risk of $O_n$, the inequality constraint is true or is nearly true. The risks of $O_{g1}$ and $O_2$ increase as $\theta_1/\sigma \to -\infty$, and approach $k$ as $\theta_1/\sigma \to \infty$. This behaviour is analogous to the behaviour of the risk of $O_1$, the inequality constrained estimator, as shown in Judge & Yancey (1981, 1986) and Thomson & Schmidt (1982). In general, the risk behaviour of $O_2$ with an unknown $\sigma^2$ is the same as in the $\sigma^2$ known case of Judge et al. (1984).

Thus far, we have focused attention on the properties of the MSIC estimator for fixed values of $c_1$ and $c_2$. Ohtani (1996) showed that the modified Stein unconstrained estimator

\begin{equation}
W_i(0_1) = \exp\left(-\theta_1^2/(2\sigma^2)\right)/(\theta_1/\sigma)^j/j!.
\end{equation}

\begin{equation}
W_i(\lambda_1) = \exp(-\lambda_1)/(\lambda_1)^j/j!.
\end{equation}

\begin{equation}
\lambda_1 = \theta_1^2/\sigma^2,
\end{equation}

\begin{equation}
h_{ij}(a, b, c, w) = C\Gamma\left(\frac{1}{2}v + c\right)\left[w + (1 - 2w)I_n(m_{ib}, v_{jc})\right],
\end{equation}

\begin{equation}
g_{ij}(a, b, c, d, w) = C\Gamma(v_c)\Gamma(m_{jid})\left[1 - w - (1 - 2w)I_n(v_c, m_{jid})\right]/\Gamma\left(\frac{1}{2}(j + a + 1 + m) + i + b\right).
\end{equation}

\begin{equation}
C = \frac{\Gamma\left(\frac{1}{2}(j + a + 1)\right)\Gamma(m_{ib})}{\Gamma(m_i)\Gamma(\frac{1}{2}v)}, \quad m_i = \frac{1}{2}m + i, \quad v_c = \frac{1}{2}v + c + d,
\end{equation}

\begin{equation}
m_{ib} = \frac{1}{2}m + i + b, \quad v_{jc} = \frac{1}{2}(v + j + a + 1) + c, \quad m_{jid} = \frac{1}{2}(m + j + a + 1 + i + b - d),
\end{equation}

\begin{equation}
\tau_1 = \frac{1}{1 + cjk/v}, \quad \tau_2 = \frac{c2m/(v + 1)}{1 + c2m/(v + 1)}.
\end{equation}

and $I_n(a, b)$ is the Incomplete Beta function.

**Proof.** The proof is available from the authors upon request.

Some features of the risk behaviour of $\hat{\theta}_{mS}$ can be observed from (3.1). First, as $\theta_1/\sigma \to \pm\infty$ or $\lambda_1 \to \infty$, $\xi \to 0$, and $R(\hat{\theta}_{mS}) \to k + ((\theta_1/\sigma)^2 - 1)\Phi(-\theta_1/\sigma) - (\theta_1/\sigma)\psi(-\theta_1/\sigma)$, which is the risk of $\hat{\theta}$, the ICLS estimator. Second, the risk of $\hat{\theta}_S$ can be obtained by fixing $c_1 = c_2 = 0$. To better understand the risk behaviour of $\hat{\theta}_{mS}$, we numerically evaluate (3.1) using the following parameter values: $\theta_1 = [-4, 4]$, $\lambda_1 = 0.1, 1, 6, 20$, $c_1 = v/(v + 2)$, $c_2 = (v + 1)/(v + 3)$, $a_1 = (k - 2)/(v + 2)$, $a_2 = (k - 2)/(v + 2)$, $a_3 = (k - 3)/(v + 3)$, $a_4 = (k - 3)/(v + 3)$ and various values of $n$ and $k$. The values of $a_1$, $a_2$, $a_3$ and $a_4$ are chosen such that they provide the minimal risk for their corresponding component estimators. To compare the risk of $\hat{\theta}_{mS}$ with that of $\hat{\theta}_S$, we define the following measure of relative efficiency: $RE(\hat{\theta}_S, \hat{\theta}_{mS}) = R(\hat{\theta}_{mS})/R(\hat{\theta}_{mS})$. The estimator $\hat{\theta}_{mS}$ has smaller risk than $\hat{\theta}_S$ if $RE(\hat{\theta}_S, \hat{\theta}_{mS}) > 1$. The higher the value of $RE(\hat{\theta}_S, \hat{\theta}_{mS})$, the more efficient is $\hat{\theta}_{mS}$ relative to $\hat{\theta}_S$. Table 1 illustrates the results for $n = 20$ and selected values of other parameters. Results involving other values of $n$ yield the same relative comparisons.

Our results indicate that at least for the cases that we have considered, $\hat{\theta}_{mS}$ is risk superior to $\hat{\theta}_S$. Other things being equal, the reduction of risk, as measured by relative efficiency, increases as $\lambda_1$ decreases. Over a large range of the parameter space, the risk for $\hat{\theta}_{mS}$ is much lower than for $\hat{\theta}_S$. Other things being equal, $RE(\hat{\theta}_S, \hat{\theta}_{mS})$ increases as $n$ decreases or $k$ increases. The above result indicates that, in practice, the MSIC estimator is particularly useful in the context of a small sample with a relatively large number of coefficients. Although Table 1 does not show the risk values of estimators, we find that both $\hat{\theta}_{mS}$ and $\hat{\theta}_S$ have risks smaller than $k$, the risk of the OLS estimator, when the inequality constraint is true or is nearly true. The risks of $\hat{\theta}_{mS}$ and $\hat{\theta}_S$ increase without bound as $\theta_1/\sigma \to -\infty$, and approach $k$ as $\theta_1/\sigma \to \infty$. This behaviour is analogous to the behaviour of the risk of $\theta$, the inequality constrained estimator, as shown in Judge & Yancey (1981, 1986) and Thomson & Schmidt (1982). In general, the risk behaviour of $\hat{\theta}_S$ with an unknown $\sigma^2$ is the same as in the $\sigma^2$ known case of Judge et al. (1984).

Thus far, we have focused attention on the properties of the MSIC estimator for fixed values of $c_1$ and $c_2$. Ohtani (1996) showed that the modified Stein unconstrained estimator
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than both $\hat{\theta}_{mS}$ and $\hat{\theta}_S$. For any given values of $\theta_1$ and $\lambda_1$, the gain in efficiency of using $\hat{\theta}^*_m$ appears to be most significant for large $k$ and relatively small $n$ values.

4. Concluding Remarks

In applied statistical analysis, information about the unknown parameters often exists in the form of an inequality constraint. We have derived and numerically evaluated the exact risk of an MSIC estimator. Over a large portion of the parameter space, the MSIC estimator has lower risk than the traditional SIC estimator. The extent of risk reduction depends, among other things, on the unobservable values of $\theta_1$ and $\lambda_1$; however, at least with reasonably small samples and a relatively large number of regressors (due to, for example, the use of seasonal dummy variables), the MSIC estimator generally leads to non-trivial reduction in estimator’s risk. It remains a topic for future research to prove the dominance of the MSIC estimator over the SIC estimator by analytical methods.

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