A Meta-model Approach to Scenario Generation in Bank Stress Testing

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Abstract: In the aftermath of the recent financial tsunami, the newly released Basel III Accord has demanded Scenario-based Stress Testing for banks. However, scenario generation is currently a bottleneck due to great heterogeneity in banking practices and organizational structures, leading to a research gap confronting IT professionals. To this end, we devise a way to treat financial scenario selection as a set-covering problem found in the field of approximation algorithms. Another ingenuity of our approach is to offering a high-level framework in order to accommodate individual bank variations, which we call as a meta-model approach. In addition, we propose a decision-support framework for scenario-based stress testing.

1. Introduction

The recent financial crisis has let regulators rethink traditional risk management methods and emphasize the role of a stress testing framework in banking risk management, as in the newly released Basel III Accord. In any stress testing frameworks, robust scenario generation techniques are crucial to the effectiveness of stress testing [1]. Compared with traditional risk management models (e.g., VaR), stress testing is forward-looking and can estimate the risk of the portfolios in a financial institution or a whole financial system under “very extreme but plausible” situations. These extreme situations are rare; but once happen, would lead to disastrous consequences [1]. A key issue in stress testing is how to construct the so-called “extreme but plausible” scenarios that can consistently reflect the original objective of stress testing.

The subjectivity in the selection of “extreme but plausible scenario” makes it a difficult job for risk managers. Another challenge for risk management experts is to choose stress scenarios from thousands of possible options which involves various risk factors and with multiple degrees of severity [2]. Some basic practice principles for plausible scenario design and calibration techniques in stress testing are proposed [2-4]. But these principles are too general to be operationalized. There is scant research specifically focusing on the scenario generation problems in banking stress testing techniques [4]. In practice, it is also reported that “the choice of stress events is frequently based on a discretionary assessment of the analyst” [5], and “constantly reviewing scenarios and looking for new ones” becomes a necessity [2].

We crystallize the core problem of scenario generation in bank stress testing as: how to generate consistent stressed scenarios to effectively test the banks’ vulnerability in extreme situations? A stress test involves a lot of stakeholders, including risk management team, senior management team, asset and liability management team, etc. [6]. They are required to collaborate closely to make a sequence of important decisions. However, there is currently a lack of IT support for
scenario-based stress testing because there is no formal method for scenario generation, thus creating a bottleneck for banks to do efficient and effective stress testing. We tackle this problem from a balanced collaborative decision-making perspective. Specifically, we first develop a high-level ontological meta-model to conceptualize the constructs in stress testing. This meta-model can be instantiated with both real-world events and experts’ judgment. Then we formulate the scenario generation procedure and propose mathematical-model-based decision support methods to assist the selection of key risk factors that constitute a realistic scenario and represent all the underlying financial shocks to be tested. A simple example is given for illustration purpose. Finally, we conclude the whole paper with discussions on the contributions, limitations, and the future work.

2. Conceptual Formulation

In this section, we present a meta-model for generating stress scenarios based on concepts found in the literature [1, 7, 8]:

- **Shocks (S):** Events that trigger the financial shocks that later will affect the financial system or specific institutions. Shocks can be those real breaking news (e.g., quake-tsunami in Japan), or can be hypothetical (e.g., based on financial expertise), or drawn from historical events, as the background of stress testing.

- **Risk Factors (R):** Risk factors represent the indicators that will directly affect the financial markets or financial institutions, such as, GDP rate, interest rate, foreign exchange rate, default rate, CDS spread, etc.

- **Scenarios:** A good scenario is a complete and consistent representation of the market events and can fully capture all the change in the relevant risk factors caused by the shocks.

Now we specify the relationships among the major components as follows (see Figure 1):

- **Shocks and Risk Factors:** Financial shocks lead to fluctuations in the risk factors. We model this kind of relationship as “shock as the sources of change in risk factors” (see Figure 1). A specific shock can affect several different risk factors. At the same time, a risk factor may also be affected by several different financial shocks. The change in risk factors caused by the shocks may be explicitly reflected, such as some important changes in the macro-economic indicators. In more cases, the impacts of financial shocks on the risk factors are implicit and need to be estimated or even predicted by experts. Moreover, the combined effects of several different shocks on the same risk factors need to be analyzed so as to eliminate the correlations among financial shocks.

- **Risk Factors and Scenarios:** A stress testing scenario consists of explicit changes in several related risk factors. We model the relationship between risk factors and scenario as attribute relationship, as shown in Figure 1. A stress testing scenario for a bank should capture all the major changes in risk factors caused by the financial shock events, which would lead to substantial increase or decrease of value in the bank holding exposures.

- **Shock-Impact-Evaluation Relationship:** Different shocks have diverse impacts on the risk factors. Moreover, most of the time, the impacts are very difficult, if not impossible, to
measure or predict. Secondly, even for a specific shock of interest, the impact of this shock on different financial institutions varies a lot. Thirdly, even for experienced domain experts, they are not fully sure their estimation is true. This is one of the sources of doubt towards the credibility of stress testing results. We represent this complicated relationship that comes from experts’ judgment as a Shock-Impact-Evaluation Relationship, i.e., an expert judgment about the impacts of a shock on risk factors with certain judgment certainty (see Figure 2). Semantically, this relationship can be represented as a triple like: \([\text{Shock}, ((\text{Risk Factor}, \text{Degree of Impact}), (\text{Risk Factor}, \text{Degree of Impact}) \ldots), \text{Certainty of Judgment}]\). The certainty of judgment represents the subjective confidence affiliated with the shock.

3. Mathematical Formulation: An Optimization Model

In the previous section, we construct a high-level ontological model for the stress testing. This formal representation defines the structure and semantics within a scenario generator. Instantiation of this meta-model produces a concrete case of scenario generation which conforms the structure and semantics defined in the meta-model. Based on this meta-model, we now present a mathematical model to provide decision support for scenario generation in stress testing systems. First we summarize several assumptions which are derived from field literature and interviews with domain experts.

**Assumption 1.** The shocks for stress testing can be defined and collected from various sources, such as banking authorities, senior managers, risk managers, advisors, etc.

**Assumption 2.** The risk factors regarding a bank’s exposure can be identified.

**Assumption 3.** Domain experts can independently distinguish and evaluate the shocks and give her/his opinions about their impacts on the risk impacts with regard to a target bank.

**Assumption 4.** The dependence among the shocks and risk factors can be addressed during the shock collection stage or considered in the testing stage, e.g., using copula techniques [9].

These assumptions are found in the literature and banking best practices. The risk factors can be identified according to the combination of bank exposures and current best practices in banking [e.g., 2, 10]. The involvement of expert judgment for probability estimation and risk management has been studied extensively [3, 9]. Based on the meta-model and these assumptions, we present next the formal definitions of the constructs in the meta-model.

**Definition 1.** Let \( S = \{S_1, S_2, \ldots, S_m\} \) be the set of shocks collected at the beginning of stress testing process, where \( m \in \mathbb{N}^+ \) is the number of shocks.
Definition 2. Let \( R = \{R_1, R_2, ..., R_n\} \) be the set of risk factors identified as the fundamental indicators related to the targeting institution, where \( n \in \mathbb{N}^+ \) is the number of risk factors.

Definition 3. Define the Shock-Impact-Evaluation relationship in the meta-model as a triple of \([S, [R - D_{-Set}], C]\), where \( S \) is a shock instance as defined in Definition 1, \( R - D_{-Set} \) is the set of risk factor and associated impact degree caused by the shock and \( C \) is the associated certainty regarding to the evaluation and \( 0 < C \leq 1 \).

Definition 4. Let \( I = \{I_1, I_2, ..., I_k\} \) be the set of all Shock-Impact-Evaluation instances, where \( k \in \mathbb{N}^+ \) is the number of these instances. As discussed above, the core problem of scenario generation is to select the most appropriate shocks that keep the “plausibility” and “credibility” of the stress testing. We derive the objective function of the model as:

\[
\begin{align*}
\text{Max} & \quad C = \prod_{i \leq i \leq k \text{ and } x_i = 1} x_i \cdot C_i \quad (1) \\
\text{s.t.} & \quad \sum_{j=1}^{n} x_i \cdot \pi_{ij} \geq 1, \ 1 \leq i \leq k \quad (2) \\
& \quad x_i \in \{0, 1\}, \ 1 \leq i \leq k \quad (3)
\end{align*}
\]

where \( x_i = 1 \) when the \( i \)-th shock is selected, otherwise \( x_i = 0 \) and \( \Pi = [\pi_{ij}] \) is a matrix in which \( \pi_{ij} = 1 \) represents the \( j \)-th risk factor is affected by the \( i \)-th shock, otherwise \( \pi_{ij} = 0 \).

The objective function tries to achieve the overall maximum certainty among the selected shocks while it is subject to the condition of covering all risk factors identified. This is actually a classic Set-Covering Problem (proof omitted due to space limit), which is both NP-hard and hard to approximate [11]. Therefore, we resort to set covering solutions to address the Scenario Generation Problem (SGP).

Due to the complexity of set-covering problem, we suggest that the choice depends on the scale of the SGP problem. For large-scale input, approximation algorithms are preferable, while for small-scale input, exact algorithms can guarantee optimal solutions [11]. The input parameters for the SGP problem can be prepared from the instantiation of the meta-model. As discussed above, the risk factors set can be identified at the early stage of the stress testing. The shocks are designed and collected collaboratively from various stakeholders. The shocks suggested or assigned by regulatory authorities can be incorporated too. The cover of risk factors by each shock and the associated weight can be computed from the Shock-Impact-Evaluation relationship. Besides, we are aware of the issue of solution uncertainty such as no solution or multiple solutions to the set-covering problem.

This meta-model-based optimization model can provide two kinds of decision support for the selection of shocks and design of scenarios. The first is on the selection of shocks to constitute a scenario. On the one hand, the optimization solutions can provide suggestions for shock selection;
on the other hand, the optimization model can be used to verify the shock selection. The second kind of decision support is on the decision of shock severity to be tested, as represented in the Shock-Impact-Evaluation Relationship. Our model can suggest and aggregate the impact evaluation of shocks from different stakeholders. Figure 3 illustrates a general framework that provides collaboration and decision support for scenario generation and stress testing.

![Figure 3. A Decision-support Framework for Bank Stress Testing](image)

### 4. An Illustrative Example

A hypothetical case is provided to illustrate our model. This example is simple and far from real cases, yet illustrates the concepts and framework of our research. In reality, there are a lot of concerns about the selection of risk factors and sources of proposed shocks, which are out of scope of this paper. Suppose we have a very small bank that attracts deposits and issue mortgages and loans to local SMEs; the only investment is in local stock market.

Now we suppose the bank identified 5 major risk factors related to its exposures to be tested in stressed scenarios: *R1 Interest rate, R2 Housing price index, R3 GDP rate, R4 CPI rate, R5 Stock Index*. Then we suppose a stress testing is proposed within the bank to investigate potential loss given possible economic downturn. Through a process of collecting and modeling shocks from different stakeholders within the bank and from regulatory authorities, 6 shocks are proposed and estimated based on expert judgment (See Table 1). We formulate this example according to the above optimization model and implement it with IBM ILOG CPLEX. Since the input scale of this case is very small, it is easy to get an optimal solution (i.e., selecting Shock 2 and Shock 4). In this way, different stakeholders involved in the stress testing can achieve a maximized certainty and acceptability about the designed stressed scenarios. At the same time, the shock degree on each risk factor can be negotiated based on the selected shocks.

<table>
<thead>
<tr>
<th>Shock</th>
<th>Short description</th>
<th>Impact on risk factors</th>
<th>Certainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>A severe local disaster, e.g., flood</td>
<td>(R3, -10%), (R4, 15%), (R5, -10%)</td>
<td>20%</td>
</tr>
<tr>
<td>S2</td>
<td>Interest rate raised 30% by central bank</td>
<td>(R1, 30%), (R2, -10%), (R4, -25%)</td>
<td>98%</td>
</tr>
<tr>
<td>S3</td>
<td>Crude oil price surges</td>
<td>(R4, 20%)</td>
<td>80%</td>
</tr>
<tr>
<td>S4</td>
<td>Local real estate market slumps</td>
<td>(R2, -30%), (R3, -15%), (R5, -30%)</td>
<td>60%</td>
</tr>
</tbody>
</table>
5. Conclusion

Our research is a first attempt towards formalizing scenario generation for bank stress testing. Our work emphasized particularly the collaboration among bank stakeholders, which improves the plausibility of the generated scenarios. We formulated the scenario generation problem into a meta-model, which led to the scenario generation process and a decision support method to assist the generation of stressed scenarios. While our current focus is on a single financial institution, the basic concepts and method in our approach can be extended and applied in the assessment of financial stress in an entire banking system. Further, there are several future research considerations, including scenario generation for stochastic stress testing, inter-dependencies among the risk factors, and second-round effect in stress testing.

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References