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Consumer Panic Buying and Quota Policy under Supply Disruptions

Biying Shou
Dept. of Management Sciences, City University of Hong Kong, biying.shou@cityu.edu.hk,

Huachun Xiong
Dept. of Mathematical Sciences, Tsinghua University, itositos@163.com,

Zuojun Max Shen
Department of Industrial Engineering and Operations Research, University of California, Berkeley, shen@ieor.berkeley.edu,

We study consumer panic buying under supply disruptions and investigate how the retailer should adjust inventory and quota policies to respond to panic buying. First, we derive a threshold for consumer product valuation above which a consumer will stockpile products for future consumption. Then, we conduct rational expectations equilibrium analysis to characterize the retailer's inventory policy and show how it is affected by consumers' risk attitude. Furthermore, we evaluate the cost of ignoring consumer behavior. Interestingly, we find that the cost is the most significant when supply reliability is intermediate (more specifically, as it approaches a *critical ratio*). Finally, we examine the effectiveness of limiting consumer purchasing quota for the retailer with limited capacity, and show when quota policy is beneficial to the retailer.

Key words: panic buying; supply disruption; strategic consumer behavior; quota policy
History:

1. Introduction

Supply disruptions, which refers to the interruption of normal product supply within supply chains, have been frequently observed around the world (Snyder et al. 2010). Disruption could be caused by various reasons, such as natural disasters, labor strikes, terrorist attacks, and changes in government regulation, etc. Supply disruption is considered as one of the top concerns for supply chain management. As a result, recent years have observed an increasing amount of research on how firms should deal with supply disruptions, e.g., via multi-sourcing, supplier certification, etc.

Little research, however, has considered the effect of supply disruption on consumer behavior. Because of the increasing access to information (e.g., via Internet and TV), many consumers also receive timely information about disruptive events. But the actual consequences of these disruptive events are often uncertain. For example, consumers may receive extreme weather warning (e.g., hurricane and snowstorm); however, whether it would actually lead to severe supply disruption is yet to know. Some consumers take active actions to reduce their potential loss due to supply disruption. Indeed, consumer panic buying associated with disruptive events has been frequently observed, where many consumers buy unusually large amounts of product to avoid possible future shortage. For example, when Hurricane Sandy stroke the New York City in 2012, many consumers rushed to store to purchase unusually large amounts of goods (Bloomberg 2012). When Hurricane Katrina disabled most of the oil drilling facilities in the U.S. Gulf coast region in 2005, consumer hoarding behavior and long lines were observed at gasoline stations (Wall Street Journal, 2005). When rice production in Australia reduced by 98% due to a long period of drought in 2008, fears of rice shortages spread around the globe. Regions such as Vietnam, India, and Hong Kong saw consumers rushing to stores and exhausting the rice supply (New York Times, 2008). Amid the earthquake and nuclear crisis in Japan in 2011, worried shoppers stripped stores of salt in Beijing, Shanghai, San Francisco, and other cities (China Daily, 2011). These consumers believed that iodized salt would protect against radiation poisoning and that there may be potential salt shortage as a result of radiation pollution.

Supply disruption, firm disruption management decisions, and consumer panic buying are closely intertwined: (1) Supply disruption can lead to consumer panic buying. Consumer panic buying can be viewed as a way to hedge supply disruption. (2) Consumer panic buying can further exaggerate the consequences of supply disruption. Abnormally high demand leads to substantial stock-outs, thus inducing more panic buying. (3) Retailer decisions (such as increasing price, limiting supply, and imposing purchasing quotas) can greatly affect consumer behaviors. Although these practices normally repress demand, they may sometimes actually increase consumer anxiety about supply shortage and make panic buying worse.

As consumer behavior and retailer decisions under supply disruption are closely related to each other, it would seem natural to study them together. However, the current studies on these aspects are largely disjoint. Consumer panic buying has been mainly studied by economists, psychologists, and sociologists who share a common focus on qualitative studies based on empirical data. Supply disruption has mainly been studied in the supply chain management literature, where strategic consumer responses are ignored.

In this paper, we aim at addressing the gap in understanding strategic consumer behavior and retailer decisions under supply disruptions. We try to provide some *rational* explanations to consumers' *seemly irrational* panic buying behavior. We also try to develop optimal strategies for retailing firms which face great challenges on both supply side (disruption) and demand side (consumer panic buying). We focus on two common disruption management strategies, safety inventory and quota policy. We also evaluate the potential loss to retailing firms if he ignores the consumer behavior changes.

Our key contributions are summarized as follows:

- First, we examine the impact of different factors on consumer decisions and identify the main drivers for consumers' panic buying behavior. We show that a consumer would stockpile product for future consumption if his valuation of the product is above a threshold. Moreover, the threshold value has monotone property with regards to the price of the product, the holding cost of the consumers, and the risk attitude of the consumers.
- Second, we use rational expectations equilibrium analysis to derive the retailer's optimal inventory policy in response to consumer panic buying. We show how the optimal level of safety inventory is dependent on the risk attitude of the consumers, the product valuations as well as the risk of supply disruption. More specifically, when consumers are risk neutral, the retailer should carry enough safety inventory to cover *all* demands in future period if both the supplier reliability and consumers' desirability of the product is lower than certain threshold values; otherwise, he will carry *no* safety inventory at all. On the other hand, when consumers are risk averse, keeping safety inventory to satisfy only *partial* of the demand is optimal under some conditions.
- Third, we investigate the cost of ignoring consumer panic buying. Interestingly, we find that the cost of ignoring consumer panic buying is the most prominent (as much as 25% of profit loss) when supply reliability is intermediate (more specifically, as it approaches a specific *critical ratio*). This result is in opposition to our intuition that the cost of ignoring consumer behaviors should be more significant when supply reliability is lower.
- Finally, we study the impact of limited capacity and the effectiveness of quota policies for the retailer. We show that limiting quota on consumer purchases benefit the retailer only when the retailer's capacity is below certain level. We also show that a quota policy is more attractive to the retailer when consumer product valuation is high, consumer holding cost is low, retailer holding cost is low, or consumer is risk averse. Furthermore, our numerical analysis demonstrates that attractiveness of the fixed quota policy increases in supply reliability and selling price when supply reliability is low, and it decreases in supply reliability and selling price when supply reliability is sufficiently high.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents our model. Section 4 analyzes consumer panic buying behavior. Section 5 derives the retailer's inventory policy. Section 6 evaluates the cost of ignoring consumer behavior change. Section 7 examines the effectiveness of a quota policy for the retailer with capacity constraint. Finally, Section 8 concludes the paper.

2. Literature review

Our work is related to the existing literature on strategic consumer behavior (e.g., Aviv and Pazgal 2008, Liu and van Ryzin 2010, van Ryzin and Liu 2008, Shen and Su 2007, Su and Zhang 2008, Su 2010, and Allon and Bassamboo 2011). Shen and Su (2007) reviews customer behavior models in the revenue management and auction literatures. Aviv and Pazgal (2008) analyzes a model with a Poisson stream of strategic customers and a single price reduction point. The authors show numerically that neglecting strategic customer behavior may lead to substantial losses. Liu and van Ryzin (2010) studies the effects of capacity rationing, and finds that competition may hinder the profitability of capacity rationing. Su (2010) studies dynamic pricing problem where consumers may stock up the product at current prices for future consumption. The author develops a solution methodology based on rational expectations, and shows that in equilibrium the seller may either charge a constant fixed price or offer periodic price promotions at predictable time intervals.

Our paper differs from the above literature as we focus on consumer panic buying behavior under supply disruptions. The behavior of a consumer who wants to avoid future supply shortages significantly differs from the behavior of a consumer seeking lower prices. Also note that our study is very different from the stream of literature on "buying frenzies" (e.g., Courty and Nasiry 2012) which examines the impact of the uncertainty of product valuations on consumer purchasing behaviors.

Our paper is also related to the research on supply uncertainty. There are three approaches to modeling supply uncertainty: random lead-time, random yield, and supply disruption. The random lead-time model assumes that the order lead-time is a random variable (e.g., Song and Zipkin 1996). In a random yield model, the realized supply level is a random function of the order/capacity level (see Yano and Lee 1995, Gerchak and Parlar 1990, Parlar and Wang 1993, Swaminathan and Shanthikumar 1999, Federgruen and Yang 2008, Kazaz 2008, Wang et al. 2010). Supply disruption models the uncertainty of supply as one of two states: "up" or "down" (e.g.,Tomlin (2006), Yang et al. (2009)). The orders are fulfilled on time and in full when the supplier is "up", and no order can be fulfilled when the supplier is "down". Snyder et al. (2010) provides a comprehensive review of the existing supply disruption research.

There are very few papers in the literature that explicitly consider the impact of supply uncertainty on customer demand. Rong et al. (2008) considers a one-shot game interaction between an unreliable supplier and multiple retailers. The retailers may inflate their order quantities in order to obtain their desired allocation from the supplier, a behavior known as the rationing game. It is shown that no Nash Equilibrium of the retailers' order quantities exists when the supplier applies the well known proportional allocation rule. Furthermore, if the retailers make reasonable assumptions about their competitors' behavior, then both the bullwhip effect and reverse bullwhip effect occur. Rong et al. (2009) investigates three different pricing strategies under supply disruption, namely naive, one-period correction (1PC), and regression approaches. The authors show that in fact the naive pricing strategy is superior to either of the more sophisticated ones, both in terms of the firm's profit and the magnitude of the customer's order variability. However, the authors do not model customer panic buying directly. Consumer panic buying is implicitly reflected by the difference between the order quantity and the underlying demand, which is assumed to be a linear function of the anticipation of price change.

To summarize, there are three key differences between our paper and the existing literature: 1) we examine the impact of different factors on consumer decisions under supply disruption, and identify the main drivers for consumers' panic buying behavior; 2) we characterize the optimal safety inventory strategy and analyze the effectiveness of quota policy, which are both common mitigation strategies in practice; 3) we demonstrate the substantial profit loss for the retailer if it ignores consumer behavior changes under supply disruption.

3. Model

We consider a monopolist retailer that sells a staple product to a mass of consumers over two periods: Periods 1 and 2. At the beginning of Period 1, the retailer orders Q_1 units of product (a decision variable) to sell, and leftover product at the end of Period 1 incurs a holding cost and is carried over for sale in Period 2 (let H be the holding cost rate per unit product per period). At the beginning of Period 2, the retailer has an opportunity to replenish its inventory. However, the replenishment at the beginning of Period 2 is not always successful due to supply disruption. We assume that supply disruption occurs with probability $1-\beta$, i.e., the retailer can successfully replenish its inventory at the beginning of Period 2 with probability β[1]. To focus on the impact of expectation of future supply uncertainty, rather than actual supply uncertainty, we only allow supply disruptions in Period 2 and not in Period 1. Without loss of generality, the procurement cost for the retailer is normalized to zero and product leftover at the end of Period 2 is assumed to have zero salvage value.

We assume that the product retail price p is exogenous and constant over the both periods. The assumption is valid in many situations. For example, during Hurricane Sandy in 2012 the State of New Jersey warned that "price gouging during a state of emergency is illegal" and that complaints would be investigated by the attorney general. Specifically, merchants are barred from raising prices more than 10 percent over their normal level during emergency conditions (Yglesias 2012). Also, during the crisis of milk powder shortage in Hong Kong, all the major milk powder providers announced publicly that the retail prices would be fixed. Furthermore, this simplifying

[1] In this paper we assume that β is common information to all players (i.e., the retailer and all consumers). We acknowledge that in reality it is possible that the retailer may be better informed of disruption risks than the consumers; and some consumers may be better informed than other consumers. The information asymmetry likely further complicates the interactions of players and exaggerates the consequences of supply disruption. To facilitate the analysis in this paper, we use this simplifying assumption and leave the information asymmetry issue to future research.

assumption of constant price also helps to isolate the impact of price changes on consumer decisions and allow us to better demonstrate the effect of supply disruptions.

There is a large population of (N) consumers, who possess heterogeneous product valuations. Each consumer valuation v is drawn from a common distribution $F(v)$. Each consumer demands at most one unit of product for consumption in each period (the utility increase for consuming extra products is assumed to be 0). At the beginning of each period, the consumers have an opportunity to purchase from the retailer. However, due to supply disruption, consumer orders may not be satisfied in Period 2, and thus a loss of consumer utility is incurred. To avoid this utility loss, a consumer may purchase 2 units in Period 1 (i.e., stockpile 1 unit for Period 2). Nevertheless, at the end of Period 1, any unused product incurs a holding cost h per unit for the consumer.

The chronology of events is as follows:

1. At the beginning of Period 1, the retailer decides how much to order initially Q_1 from its supplier. This order will be received in full by the retailer immediately.

2. Each consumer decides his purchase quantity $z_1 \in \{0, 1, 2\}$ in Period 1.

3. During Period 1, each consumer uses $min\{1, z_1\}$ units of product. The leftover product, if any, incurs a holding cost.

4. At the beginning of Period 2, the retailer decides how much to order Q_2 from its supplier. Due to supply disruption, this second order may not be fulfilled.

5. Each consumer decides his purchase quantity $z_2 \in \{0, 1\}$ in Period 2. However, fulfillment of these orders depends on the availability of retailer inventory. In particular, we assume the fulfillment rate at the beginning of Period 2 is α, which depends on the probability of supply disruption and the retailer's inventory decision. We further assume the all consumers have rational expectations on this fulfillment rate, which is a common assumption in the existing literature (e.g., Liu and Ryzin 2008, Su and Zhang 2009).

6. During Period 2, consumers consume any on hand product.

We assume the retailer is risk neutral, since in general retailers have large and well-diversified product portfolios. We assume that consumers are risk averse (the risk neutral case is considered as a special case). In particular, we assume a consumer with valuation v has a utility function $U_v(x) = U(x - r_v)$, where $U(\cdot)$ is an increasing concave function with $U(0) = 0$ (the risk neutral case is a special case with $U(x) = x$). We also assume that

$$\lim_{x \to +\infty} \frac{U(x-y)}{U(x)} = 1, \tag{1}$$

for any real number y. Many increasing concave utility functions satisfy Eqn. (1), such as $U(x) = 1 - exp(-rx) \;\; (0 < r < +\infty)$, $U(x) = x^\gamma \;\; (0 < \gamma \le 1)$, etc. In $U_v(\cdot)$, r_v is the reference point (please see Prospect Theory of Kahneman and Tversky 1979 for details). We assume that

$$r_v = \begin{cases} v - p, & if \quad v > p, \\ 0, & if \quad v \le p. \end{cases} \tag{2}$$

Table 1 Notation

H (h)	Retailer's (consumers') holding cost rate per unit per period
p	Regular selling price of the product
N	Number of consumers
v	A consumer's valuation to the product
$F(v)$	Distribution of consumer valuation
$\overline{v}$	The highest valuation when $F(v)$ is assumed to be uniform distribution
S	Supply status; $S=0$ means supply disruption while $S=1$ means full order fulfillment
β	Supply reliability (the probability of $S=1$)
z_1 (z_2)	A consumer's purchasing quantity in Period 1 (Period 2)
Q_1 (Q_2)	The retailer's order quantity in Period 1 (Period 2)
Q_1^* (Q_2^*)	The retailer's equilibrium order quantity in Period 1 (Period 2)
$U_v(\cdot)$	The utility function of a consumer with valuation v
r_v	The reference point of a consumer with valuation v
γ	A parameter measures consumers' degree of risk aversion
α (α_c, α^*)	The fulfill rate (consumers' belief on the fulfill rate, the equilibrium fulfill rate) in Period 2
$u(\cdot)$	A consumer's expected utility
$T(p,h,\alpha_c)$	The threshold of consumer valuation v for panic buying
D_1	Total consumer demand in Period 1
D_2	Total consumer demand in Period 2
$\Pi(\cdot)$ Π^*	The retailer's (optimal) expected profit under the basic model
Q_{i1} (Q_{i2})	The retailer's order quantity at the beginning of Period 1 (Period 2) under Case i
$\Pi_i(\cdot)$ Π_i^*	The retailer's (optimal) expected profit under Case i
	$i=I$: the case of ignoring consumer behaviors; $i=f$: the case of fixed quota;
PL	The retailer's profit loss of ignoring consumer behaviors
K	The retailer's capacity
δ	The retailer's capacity over demand of Period 1
θ	The percentage of Type-2 consumers served in Period 1 with uniform arrival assumption
x	The yield proportion in the random proportional yield model
μ	The mean of x, $E(x)=\mu$
$B(x)$	The distribution of x

All of these parameters and distribution functions are common knowledge to the retailer and consumers. We define the notation $\overline{F}(x)=1-F(x)$ and $x^+=max\{x,0\}$.

Table 1 provides a summary of the notation in this paper.

4. Consumers' purchasing decisions

In this section we analyze consumers' panic buying behavior. For a given belief about the fulfill rate in Period 2, we derive consumers' optimal purchasing decisions in each period. Note that based on the rational expectations assumption, all of the consumers' beliefs are the same and consistent with the realized fulfillment rate in equilibrium. Hence, we only need to consider the case where consumers have homogeneous beliefs over the fulfillment rate.

Let α_c be the consumers' belief about the fulfillment rate and S be a random variable indicating whether a consumer receives his order in Period 2. $S=1$ means that the consumer receives his order in Period 2 and $S=0$ means he does not. Consider a consumer with valuation v. Recall that each consumer demands at most one unit of product for consumption during each period. Clearly, if $v \le p$, then the consumer will not buy any product in Periods 1 or 2. When $v > p$, a consumer's purchasing quantity at the beginning of Period 1, z_1, is either 1 or 2. If he purchases one unit in Period 1, then he will also purchase one unit in Period 2; if he purchases two units in Period 1, then he will not purchase anything in Period 2. In other words, his purchasing quantity at the beginning of Period 2, z_2, equals $2 - z_1$. The consumer's payoff is

$$\pi = \begin{cases} 2(v-p) - h(z_1 - 1), & if \quad S = 1, \\ (v-p)z_1 - h(z_1 - 1), & if \quad S = 0. \end{cases} \tag{3}$$

In the case of $S = 1$, since the consumer gets what he purchases in both periods, the utility from consumption is always $2(v-p)$ regardless of z_1. In the case of $S = 0$, the utility from consumption is $(v-p)z_1$. The term $h(z_1 - 1)$ is the consumer's holding cost. For a given belief about the fulfillment rate α_c, the consumer's expected utility with z_1 and $z_2 = 2 - z_1$ is

$$u(z_1) = E_S U_v(\pi - r_v) = \alpha_c U\Big((v-p) - h(z_1 - 1)\Big) + (1 - \alpha_c) U\Big((v - p - h)(z_1 - 1)\Big). \tag{4}$$

We identify the optimal purchasing policy for consumers in the following theorem.

THEOREM 1. *For a given belief about the fulfill rate in Period 2 α_c, there exists a threshold $T(p, h, \alpha_c) \ge p + h$ with*

$$\frac{U(T(p,h,\alpha_c) - p - h)}{U(T(p,h,\alpha_c) - p)} = \alpha_c, \tag{5}$$

such that the optimal purchasing policy for the consumer with valuation v is as follows:

1. *If $v \le p$, then $z_1^*(\alpha_c) = z_2^*(\alpha_c) = 0$;*
2. *If $p < v \le T(p,h,\alpha_c)$, then $z_1^*(\alpha_c) = z_2^*(\alpha_c) = 1$;*
3. *If $v > T(p,h,\alpha_c)$, then $z_1^*(\alpha_c) = 2,\ z_2^*(\alpha_c) = 0$.*

Proof. Case 1 is straightforward, so we focus on the proofs of Cases 2 and 3. By Eqn. (4), the consumer's expected utility when purchasing one unit in both periods and two units in Period 1 only are respectively

$$u(1) = \alpha_c U(v-p),$$
$$u(2) = U(v-p-h).$$

Thus, $u(1) > (<) u(2)$ is equivalent to

$$\frac{U(v-p-h)}{U(v-p)} < (>) \alpha_c.$$

Let $g(v) = U(v-p-h)/U(v-p)$. $g(v)$ is continuous and monotonically increasing in v since $U(x)$ is an increasing concave function. In addition, we have $g(p+h)=0$ and $lim_{v\to+\infty} g(v)=1$ by Assumption (1). Therefore, there exists a number $T(p,h,\alpha_c) \geq p+h$ with

$$\frac{U(T(p,h,\alpha_c)-p-h)}{U(T(p,h,\alpha_c)-p)} = \alpha_c,$$

such that the consumer will purchase two units in Period 1 if $v > T(p,h,\alpha_c)$, and he will purchase one unit during each period if $p < v \leq T(p,h,\alpha_c)$. ■

Theorem 1 identifies a threshold policy for consumers, which is quite intuitive. When the consumer valuation is low, he will not buy any product. When the consumer valuation is intermediate, he will only buy one unit product in each period if available. When the consumer valuation is high, he will stockpile one unit of product in Period 1. Next we examine some properties of the threshold $T(p,h,\alpha_c)$. The proofs of the following two propositions are found in the appendix.

PROPOSITION 1. *$T(p,h,\alpha_c)$ is increasing in p,h and α_c.*

Proposition 1 indicates that consumers are more likely to stockpile product when the selling price is low, the consumer holding cost is low, or consumer confidence about availability in Period 2 is low.

Next we investigate how the threshold $T(p,h,\alpha_c)$ changes with the degree of consumer risk aversion. According to Pratt (1964), an increase in the degree of risk aversion can be represented by an increasing concave transformation. Let $W(\cdot) = \varphi\big(U(\cdot)\big)$, where φ is an increasing concave function with $\varphi(0) = 0$. This utility function $W(\cdot)$ represents a higher degree of risk aversion. Let T_U, T_W be the thresholds corresponding to $U(\cdot)$, $W(\cdot)$ respectively. We obtain the following proposition.

PROPOSITION 2. *$T_U \geq T_W$, that is, a consumer with a higher degree of risk aversion is more likely to purchase 2 units at the beginning of Period 1.*

Proposition 2 implies that more risk averse consumers are more likely to stockpile product.

5. Retailer's Optimal Inventory Policy

We conduct equilibrium analysis in this section to study the retailer's optimal inventory policy and the cost of ignoring consumer panic buying behavior. To study strategic interactions between the retailer and consumers, and strategic interactions among the consumers, we will adopt the framework of rational expectations (RE) equilibrium. This concept of RE was originally proposed by John F. Muth (1961) and have been widely used in economics models to study how a large number of individuals, firms and organizations make choices under uncertainty. The basic idea of RE is that the outcomes of the model do not differ systematically from what players in the model expected them to be. The RE concept is also commonly used in the recent emerging strategic consumer behavior literature in operations management (e.g., Liu and Ryzin 2008, Su and Zhang 2009). The implication of RE in our model will be discussed in Definition 1.

From Theorem 1, we know that given fulfillment rate belief α_c, the total consumer demand in Period 1 is

$$D_1 = N\overline{F}(p) - N\overline{F}\big(T(p,h,\alpha_c)\big) + 2N\overline{F}\big(T(p,h,\alpha_c) = N\overline{F}(p) + N\overline{F}\big(T(p,h,\alpha_c)\big), \tag{6}$$

and in Period 2 is

$$D_2 = N\overline{F}(p) - N\overline{F}\big(T(p,h,\alpha_c)\big). \tag{7}$$

Thus, the retailer's expected profit from ordering $Q_1(Q_2)$ during period 1(2) is

$$\begin{aligned}\Pi(Q_1,Q_2,\alpha_c) &= p\min\{D_1,Q_1\} - H\big[Q_1 - D_1\big]^+ \\ &\quad + \beta p\min\{Q_2 + \big[Q_1 - D_1\big]^+, D_2\} + (1-\beta)p\big[Q_1 - D_1\big]^+,\end{aligned} \tag{8}$$

where β is the probability that the supplier is up in Period 2 and $T(p,h,\alpha_c)$ is defined in (5). In Eqn. (8), the first term represents the revenue in Period 1, the second term represents the holding cost, the third term represents the expected revenue when the supplier is up in Period 2, and the last term represents the expected revenue when the supplier is down in Period 2. For any given α_c, let $Q_1^*(\alpha_c)$, $Q_2^*(\alpha_c)$ be the optimal order quantities that maximize (8). Because the total consumer demand over both periods is $2N\overline{F}(p)$, if $D_1 > Q_1^*(\alpha_c)$ then the retailer should order $Q_2^*(\alpha_c) = 2N\overline{F}(p) - D_1$, and if $D_1 \le Q_1^*(\alpha_c)$ then the retailer should order $Q_2^*(\alpha_c) = 2N\overline{F}(p) - Q_1^*(\alpha_c)$. In other words, $Q_2^*(\alpha_c) = 2N\overline{F}(p) - \max\{D_1, Q_1^*(\alpha_c)\}$. Based on this reasoning, we omit the decision Q_2 and only consider the decision Q_1 for the rest of this paper. Next we introduce the definition of a RE equilibrium.

Definition 1. A RE equilibrium satisfies the following conditions:

(i) Given belief α_c, the optimal purchasing policy for a consumer with valuation v is $z_1 = z_1^*(\alpha_c)$, $z_2 = z_2^*(\alpha_c)$, where $z_1^*(\alpha_c)$ and $z_2^*(\alpha_c)$ are defined in Theorem 1.

(ii) Given belief α_c, the retailer's optimal order quantities satisfy $Q_1 = Q_1^*(\alpha_c)$ and $Q_2 = Q_2^*(\alpha_c) = 2N\overline{F}(p) - \max\{D_1, Q_1^*(\alpha_c)\}$.

(iii) The beliefs of all consumers α_c are consistent with the realized fulfillment rate α, i.e.,

$$\alpha_c = \alpha = \beta + (1-\beta)\min\left\{\frac{\big[Q_1^*(\alpha_c) - D_1\big]^+}{D_2}, 1\right\}. \tag{9}$$

In Eqn. (9), the term $\Big[Q_1^*(\alpha_c) - D_1\Big]^+$ represents the retailer's inventory at the beginning of Period 2, so the ratio between this term and the total demand in Period 2 is the fulfillment rate when a supply disruption occurs. Since $\alpha_c = \alpha$ at the equilibrium, we use α to replace α_c hereafter. Furthermore, we write $Q_1^*(\alpha_c)$ as Q_1^* for simplicity.

Next we describe the retailer's inventory policy in a RE equilibrium. First, we show that the retailer's initial order quantity Q_1^* is not greater than $2N\overline{F}(p)$, the total demand of both periods. Second, we show that the retailer's initial order quantity Q_1^* should be greater than or equal to D_1, the total demand in Period 1. Thus, the retailer must choose $Q_1^* \in \left[D_1, 2N\overline{F}(p)\right]$ to maximize profit. We see that the RE equilibrium can be computed by solving the following optimization problem:

$$\max_{Q_1,T,\alpha} \ \Pi = 2pN\overline{F}(p)\beta + pQ_1(1-\beta) - H[Q_1 - D_1] \tag{10}$$

$$s.t. \ \ \alpha = \beta + (1-\beta)\min\left\{\frac{\left[Q_1 - D_1\right]^+}{D_2}, 1\right\}, \tag{11}$$

$$\alpha U(T-p) = U(T-p-h), \tag{12}$$

$$Q_1 \in [D_1, 2N\overline{F}(p)]. \tag{13}$$

The objective function (10) is the simplified form of the retailer's expected profit in Eqn. (8) based on the choices $Q_1 \in [D_1, 2N\overline{F}(p)]$ and $Q_2 = 2N\overline{F}(p) - Q_1$. Constraint (11) follows from Eqn. (9) of the RE equilibrium definition, constraint (12) comes from Eqn.(5), the definition of the threshold T, and constraint (13) is due to the discussion before Eqn. (10). We omit the constraint $T \in [T(p,h,\beta), +\infty)$ because it is implied by constraints (11)-(13). Substituting (11) and (12) into the objective function (10), we have the following problem:

$$\max_{T} \ \ \Pi(T) = 2pN\overline{F}(p) - N\left[p - \frac{H\beta}{1-\beta} - \left(p - \frac{H}{1-\beta}\right)\frac{U(T-p-h)}{U(T-p)}\right]\cdot[F(T) - F(p)] \tag{14}$$

$$s.t. \ \ T \in \big[T(p,h,\beta), +\infty\big). \tag{15}$$

Once the optimal solution to Problem (14)-(15), T^*, is obtained, we can easily calculate the realized fulfillment rate and the optimal order quantity Q_1^*:

$$\alpha^* = U(T^* - p - h)/U(T^* - p), \tag{16}$$

$$Q_1^* = \frac{\alpha^* - \beta}{1-\beta}N[F(T^*) - F(p)] + N[\overline{F}(T^*) + \overline{F}(p)]. \tag{17}$$

We will assume for the rest of this paper that consumers possess a power utility function $U(x) = x^\gamma$ $(0 < \gamma \le 1)$, which is a common utility function representing risk aversion in economics and operations management literature (e.g., Liu and Ryzin 2008). A lower value of γ represents a higher degree of risk aversion. In addition, we assume consumer valuations are uniformly distributed on $[0, \overline{v}]$, where the average consumer valuation is $\overline{v}/2$. The quantitity $\overline{v}$ can be interpreted as a measure of consumers' desirability for the product. To avoid trivial cases, we assume that $\overline{v} > p$. Based on Eqn.(5), we have $T(p,h,\alpha) = p + h/(1-\alpha^{\frac{1}{\gamma}})$, and Problem (14)-(15) becomes

$$\max_T \quad \Pi(T) = \frac{2Np(\overline{v}-p)}{\overline{v}} - N \cdot \left\{ \min\left(\frac{T}{\overline{v}}, 1\right) - \frac{p}{\overline{v}} \right\} \cdot \left\{ H + \left(p - \frac{H}{1-\beta}\right)\left[1 - \left(1 - \frac{h}{T-p}\right)^{\gamma}\right] \right\} \quad (18)$$

$$s.t. \quad T \in \left[p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}, +\infty\right). \quad (19)$$

There are two interesting questions: (1) Under what conditions should the retailer carry inventory at the end of Period 1 to increase the fulfillment rate in Period 2? If so, how much inventory should the retailer carry over? (2) How do consumer risk preferences affect the retailer's inventory decision? The following two theorems answer these questions.

THEOREM 2. *Suppose consumers are risk neutral, i.e., $\gamma = 1$. The retailer's optimal order quantity Q_1^* and the realized fulfillment rate α^* are*
(i) $Q_1^* = N[\overline{F}(p) + \overline{F}(p + \frac{h}{1-\beta})]$, $\alpha^* = \beta$, if $(\beta, \overline{v}) \in \Omega_1^n$;
(ii) $Q_1^* = 2N\overline{F}(p)$, $\alpha^* = 1$, if $(\beta, \overline{v}) \in \Omega_2^n$,
where $\Omega = \{(\beta,\overline{v}) | 0 \le \beta \le 1, p \le \overline{v} \le +\infty\}$, $\Omega_2^n = \{(\beta,\overline{v}) | \beta < 1 - \frac{H}{p}, \overline{v} < \frac{p(H+h)}{H}\}$ and $\Omega_1^n = \Omega - \Omega_2^n$.

Proof. We consider two cases: $\beta > 1 - H/p$ and $\beta \le 1 - H/p$.

Case (a): $\beta > 1 - H/p$. In this case, we have $p < H/(1-\beta)$. It can be verified that $\Pi(T)$ in (18) is decreasing with respect to T, so it is optimal to choose $T^* = T(p,h,\beta)$. Thus $\alpha^* = \beta$ and $Q_1^* = N[\overline{F}(T(p,h,\beta)) + \overline{F}(p)] = N[2 - \frac{2p}{\overline{v}} - \frac{h}{(1-\beta)\overline{v}}]$.

Case (b): $\beta \le 1 - H/p$. Problem (18)-(19) can be split into two subproblems:

$$\max_T \quad \Pi_1(T) = \frac{2Np(\overline{v}-p)}{\overline{v}} - \frac{N(\overline{v}-p)}{\overline{v}} \cdot \left\{ H + \left(p - \frac{H}{1-\beta}\right)\left[1 - (1 - \frac{h}{T-p})^{\gamma}\right] \right\} \quad (20)$$

$$s.t. \quad T \in [\overline{v}, +\infty), \quad (21)$$

$$\max_T \quad \Pi_2(T) = \frac{2Np(\overline{v}-p)}{\overline{v}} - \frac{N(T-p)}{\overline{v}} \cdot \left\{ H + \left(p - \frac{H}{1-\beta}\right)\left[1 - \left(1 - \frac{h}{T-p}\right)^{\gamma}\right] \right\} \quad (22)$$

$$s.t. \quad T \in \left[p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}, \overline{v}\right]. \quad (23)$$

First consider the optimal solution of Subproblem (20)-(21). It is clear that $\Pi_1(T)$ as defined in (21) is increasing with respect to T. Thus, the optimal solution to Problem (20)-(21) is $T_1^* = +\infty$ and the optimal value is

$$\Pi_1^* = \Pi_1(T_1^*) = \frac{2Np(\overline{v}-p)}{\overline{v}} - \frac{NH(\overline{v}-p)}{\overline{v}}. \quad (24)$$

Now consider Subproblem (22)-(23). Taking first and second derivatives of $\Pi_2'(T)$ in (22), we obtain

$$\Pi_2'(T) = -\frac{N}{\overline{v}} \left\{ H - \left(p - \frac{H}{1-\beta}\right)\left[\left(1 - \frac{h}{T-p}\right)^{\gamma-1}\left(1 - \frac{(1-\gamma)h}{T-p}\right) - 1\right] \right\}. \quad (25)$$

and $\Pi_2''(T) < 0$. Since $\gamma = 1$, we have $\Pi_2'(T) < 0$ for all $T > p + h$, which means that the optimal solution of Subproblem (22)-(23) is $T_2^* = p + \frac{h}{1-\beta}$, and the optimal value is

$$\Pi_2^* = \Pi_2(T_2^*) = \frac{2Np(\overline{v} - p)}{\overline{v}} - \frac{Nph}{\overline{v}}. \tag{26}$$

If $p \leq \overline{v} < p + \frac{h}{1-\beta}$, then Subproblem (22)-(23) vanishes and the optimal solution is $T^* = +\infty$. If $p + \frac{h}{1-\beta} \leq \overline{v} < \frac{p(H+h)}{H}$, then $\Pi_1^* > \Pi_2^*$, which implies $T^* = T_1^* = +\infty$. If $\overline{v} \geq \frac{p(H+h)}{H}$, then $\Pi_1^* \leq \Pi_2^*$ and $T^* = T_2^* = p + \frac{h}{1-\beta}$. ■

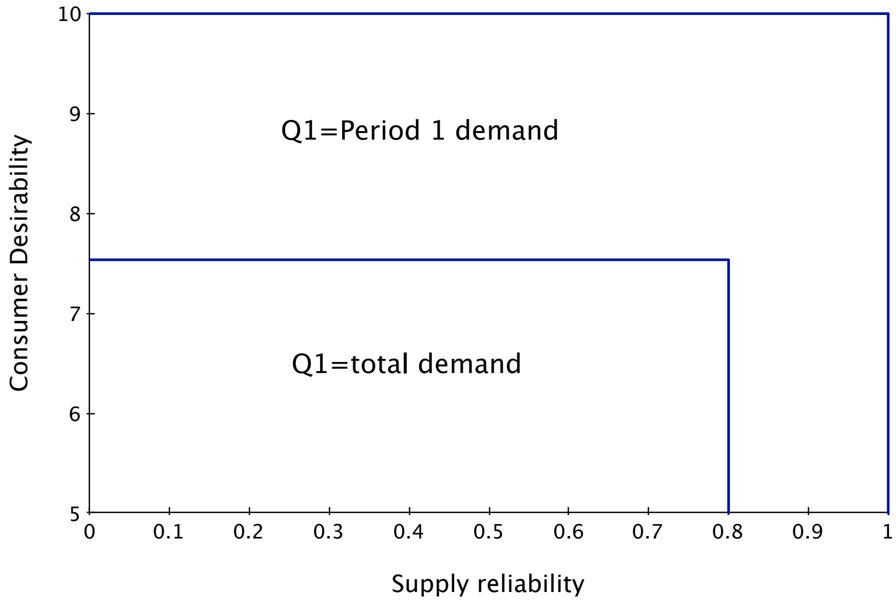
Theorem 2 describes the retailer's optimal policy when facing risk neutral consumers: when both the supply reliability and consumer product valuations are lower than certain thresholds, the retailer should order enough inventory in period 1 to cover all demands in two periods. Otherwise, it will just order inventory to satisfy period 1 demand and carry no extra inventory over to period 2 at all. Fig. 1 provides an illustration with retail price $p = 5$, the retailer's holding cost $H = 1$, and the consumers' holding cost $h = 0.5$. The X-axis is supply reliability β and the Y-axis is maximum consumer product valuation $\overline{v}$, which reflects consumer desirability for the product. The two-dimensional area of $(\beta, \overline{v})$ is divided into two regions. In the lower left region, the retailer will carry inventory from Period 1 to fulfill all consumer demand in Period 2. In the other region, the retailer will carry no inventory from Period 1 to Period 2.

The intuition of Theorem 2 is as follows. First, when the supplier reliability β is high, the retailer is most likely to receive his second inventory order to fulfill consumer demand in Period 2, thus it does not have incentive to carry extra inventory. Second, when the supplier reliability β is low but the consumers' valuation of the product is high, more consumers will choose to stockpile inventory in Period 1, which again provides no incentive for the retailer to carry safety inventory into Period 2. It is when both the supplier reliability β and the consumers' valuation of the product is low, will the retailer choose to carry safety inventory. Moreover, since in such case the expected stockout cost always outweighs the retailer's holding cost, the retailer will carry enough safety inventory to meet *all* demands in Period 2.

The following proposition shows that the retailer should not carry inventory when supply reliability is high, even under only mild conditions on the distribution of consumer valuation and the consumer utility functions.

PROPOSITION 3. *For any consumer valuation distribution $F(v)$ and any increasing concave utility function $U(x)$, if $(1-\beta)p < H$ then the optimal order quantity and fulfillment rate are $Q_1^* = N[\overline{F}(T(p,h,\beta)) + \overline{F}(p)]$ and $\alpha^* = \beta$.*

Figure 1 The optimal inventory policy for the retailer facing risk neutral consumers



Next we discuss the retailer's optimal order quantity when consumers are risk averse. Before stating Theorem 3, we introduce some more notation. Let β_t be the solution of

$$H-(p-\frac{H}{1-\beta})[\gamma\beta^{1-\frac{1}{\gamma}}+(1-\gamma)\beta-1]=0. \tag{27}$$

Let T^o be the solution of $\Pi_2^{'}(T)=0$ in (25), i.e.,

$$H-\Big(p-\frac{H}{1-\beta}\Big)\Big[\Big(1-\frac{h}{T-p}\Big)^{\gamma-1}\Big(1-\frac{(1-\gamma)h}{T-p}\Big)-1\Big]=0. \tag{28}$$

Denote

$$v_t(\beta)=\begin{cases} p+\frac{T^o-p}{H}\{H+(p-\frac{H}{1-\beta})[1-(1-\frac{h}{T^o-p})^\gamma]\}, & if\ \beta<\beta_t, \\ p+\frac{ph(1-\beta)}{H(1-\beta^{\frac{1}{\gamma}})}, & if\ \beta\geq\beta_t, \end{cases} \tag{29}$$

and

$$\Omega=\Big\{(\beta,\overline{v})|0\leq\beta\leq 1, p\leq\overline{v}\leq+\infty\Big\},$$
$$\Omega_2^v=\Big\{(\beta,\overline{v})|0\leq\beta<1-\frac{H}{p},\overline{v}<v_t(\beta)\Big\},$$
$$\Omega_3^v=\Big\{(\beta,\overline{v})|0\leq\beta\leq\beta_t,\overline{v}>v_t(\beta)\Big\},$$
$$\Omega_1^v=\Omega-\big(\Omega_2^v\cup\Omega_3^v\big).$$

THEOREM 3. *Suppose the consumers are risk averse with $0<\gamma<1$. The retailer's optimal order quantity and fulfillment rate follow:*

(i) If $(\beta,\overline{v})\in\Omega_1^v$, then $Q_1^*=N\Big[\overline{F}(p)+\overline{F}\big(p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}\big)\Big]$, $\alpha^*=\beta$;

(ii) If $(\beta,\overline{v})\in\Omega_2^v$, then $Q_1^*=2N\overline{F}(p)$, $\alpha^*=1$;

(iii) If $(\beta,\overline{v})\in\Omega_3^v$*, then*

$$\begin{aligned}Q_1^* &= \frac{\alpha^*-\beta}{1-\beta}N\left[F(T^o)-F(p)\right]+N\left[\overline{F}(T^o)+\overline{F}(p)\right]\\ &\in \Big(N\Big[\overline{F}(p)+\overline{F}(p+\frac{h}{1-\beta^{\frac{1}{\gamma}}})\Big],2N\overline{F}(p)\Big),\\ \alpha^* &= \frac{(T^o-p)^\gamma}{(T^o-p-h)^\gamma}\in(\beta,1).\end{aligned}$$

Proof. There are three cases: (a) $\beta>1-\frac{H}{p}$, (b) $\beta_t<\beta\le 1-\frac{H}{p}$, (c) $0<\beta\le\beta_t$.

For Case (a), we have $Q_1^*=N[\overline{F}(T(p,h,\beta))+\overline{F}(p)]$ and $\alpha^*=\beta$ by Proposition 3.

For Case (b), we have $\beta>\beta_t$ and

$$X(\beta,\gamma,p,H)=\Pi^{'}(p+\frac{h}{1-\beta^{\frac{1}{\gamma}}})=-\frac{N}{\overline{v}}\{H-(p-\frac{H}{1-\beta})[\gamma\beta^{1-\frac{1}{\gamma}}+(1-\gamma)\beta-1]\}<0, \tag{30}$$

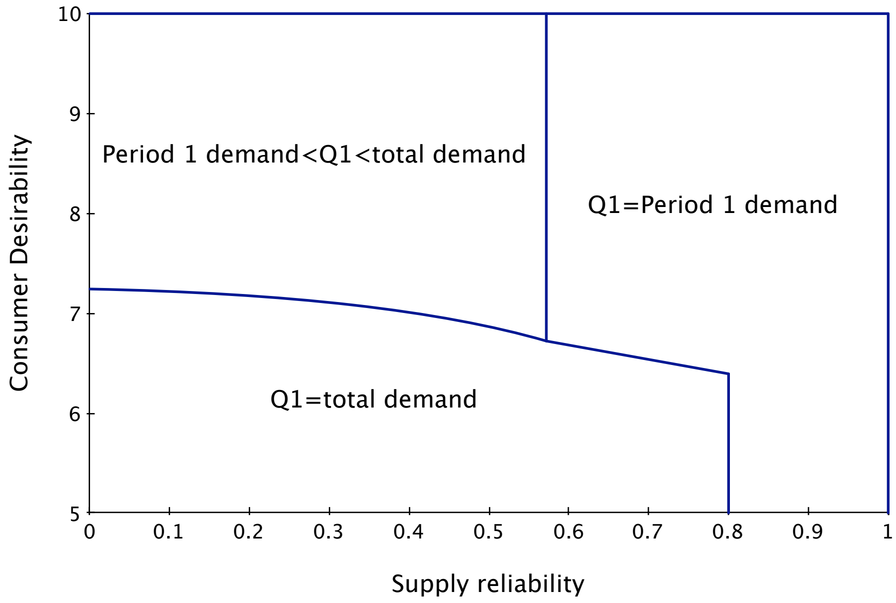
we have $T^o<p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}$. (It can be verified that $X(\beta,\gamma,p,H)$ is decreasing with respect to β, $lim_{\beta\to 0}X(\beta,\gamma,p,H)=+\infty$ and $X(1-H/p,\gamma,p,H)=-NH/\overline{v}<0$. Thus, β_t is in $(0,1-\frac{H}{p})$.) Recall that Problem (18)-(19) can be split into Subproblems (20)-(21) and (22)-(23). By a similar argument to the proof of Theorem 2, the optimal solution to Subproblem (20)-(21) is $T_1^*=+\infty$ and the optimal value is Π_1^* in (24). The optimal solution to Subproblem (22)-(23) is $T_2^*=p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}$ and the optimal value is $\Pi_2^*=\Pi_2(T_2^*)=\frac{2Np(\overline{v}-p)}{\overline{v}}-\frac{Nph(1-\beta)}{\overline{v}(1-\beta^{\frac{1}{\gamma}})}$.

If $\overline{v}<p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}$, then Subproblem (22)-(23) vanishes and the optimal solution is $T^*=T_1^*=+\infty$. If $p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}\le\overline{v}<v_t(\beta)$, then $\Pi_1^*>\Pi_2^*$, which indicates $T^*=T_1^*=+\infty$. If $\overline{v}\ge v_t(\beta)$, then $\Pi_1^*\le\Pi_2^*$, which implies that $T^*=T_2^*=p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}$.

In Case (c), it can be verified that $T^o\ge p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}$. The optimal solution of Subproblem (22)-(23) becomes $T_2^*=min\{T^o,\overline{v}\}$. If $\overline{v}<T^o$, then $T_2^*=\overline{v}$. Thus, $\Pi_1^*=\Pi_1(+\infty)>\Pi_1(\overline{v})=\Pi_2(\overline{v})=\Pi_2^*$, which indicates that $T^*=T_1^*=+\infty$. If $T^o\le\overline{v}<v_t(\beta)$, then $T_2^*=T^o$ (implied by $T^o\le\overline{v}$) and $\Pi_1^*=\Pi_1(+\infty)>\Pi_2(T^o)=\Pi_2^*$ (implied by $\overline{v}<v_t(\beta)$). Thus, $T^*=T_1^*=+\infty$. If $\overline{v}\ge v_t(\beta)$, then $T_2^*=T^o$ and $\Pi_1^*=\Pi_1(+\infty)\le\Pi_2(T^o)=\Pi_2^*$. Thus, $T^*=T_2^*=T^o$. ■

Theorem 3 describes the optimal inventory policy for risk averse consumers: when the supply reliability is high, the retailer should not carry inventory at the end of Period 1; when the supply reliability is low and consumer product valuations are low, the retailer should carry inventory at the end of Period 1 to cover all demand in Period 2; otherwise, it should carry inventory to cover partial demand in Period 2. Fig. 2 displays a numerical example with retail price $p=5$, retailer holding cost $H=1$, consumer holding cost $h=0.5$, and consumer risk aversion measure $\gamma=0.5$. In Fig. 2, the two-dimensional area of $(\beta,\overline{v})$ is divided into 3 regions. In the lower-left region, the retailer carries inventory at the end of Period 1 to cover all the demand in Period 2 and the realized fulfillment rate in Period 2 is 1. In the upper-left region, the retailer carries some inventory, but

Figure 2 The optimal inventory policy for the retailer facing risk averse consumers



does not cover all demand in Period 2, so the realized fulfillment rate in Period 2 is between β and 1. In the right region, the retailer does not carry any inventory at the end of Period 1 and the realized fulfillment rate in Period 2 is β.

The interpretation of this result is as follows. The retailer considers the trade-off between two types of costs when making ordering decisions: the inventory holding cost and the stockout cost caused by supply disruption. If the retailer increases the inventory at the end of Period 1 by one unit, then it incurs extra holding cost H but reduces stockout cost p with probability $(1-\beta)$. Furthermore, increasing the inventory at the end of Period 1 will also increase the fulfillment rate α in Period 2, which decreases the number of consumers who purchase two units of product in Period 1. Hence, increasing inventory may increase the retailer's holding cost by more than H per unit. If the supplier reliability is high, the reduced stockout cost by carrying one unit of inventory $(1-\beta)p$ is less than the increased holding cost. Thus, the retailer has no incentive to carry inventory at the end of Period 1. Now we consider low supply reliability and high stockout cost. When consumer desirability for the product is low, not many consumers purchase two units of product in Period 1. Even if the retailer carries inventory to cover all demand in Period 2 (which increases the fulfillment rate in Period 2 to 1), the demand of Period 1 does not significantly decrease. The extra holding cost due to reducing consumer panic buying incentives and shrinking the demand in Period 1 is relatively small. Hence, the retailer carries inventory at the end of Period 1 to cover all demand in Period 2. When consumer desirability for the product is high, there are a lot of consumers panic buying in Period 1. The demand shrinks more dramatically and the holding cost increases more significantly when the inventory at the end of Period 1 is greater. There is a balance point of inventory at the end of Period 1, where the reduced stockout cost is equal to the marginal holding cost. Therefore the retailer tends to carry less inventory to cover partial demand in Period 2 when the consumer desirability for the product is high.

Comparing Theorems 2 and 3, we see that consumers' risk preferences influence the retailer ordering decisions. Interestingly, when supply reliability is low and consumer desirability for the product is high, the retailer does not carry any safety inventory if consumers are risk neutral but it does carry safety inventory if consumers are risk averse.

6. Cost of ignoring consumer behaviors

In this section we answer the following question: when will ignoring consumer behaviors have severe consequences for the retailer? To evaluate the cost of ignoring consumer behaviors, we first derive the optimal order quantity when the retailer ignores consumer panic buying. With this assumption, the retailer chooses Q_{I1} and Q_{I2} (the subscript I represents the case of ignoring consumer behavior) to maximize the expected profit

$$\Pi_I(Q_{I1}, Q_{I2}) = pQ_{I1} - H[Q_{I1} - N\overline{F}(p)] + \beta p Q_{I2}. \tag{31}$$

Clearly, for any given Q_{I1}, it is optimal to order $Q_{I2} = 2N\overline{F}(p) - Q_{I1}$. If we replace Q_{I2} by $2N\overline{F}(p) - Q_{I1}$, the objective function in Eqn. (31) is linear in Q_{I1}. Thus the optimal initial order quantity is $Q^*_{I1} = N\overline{F}(p)$, if $\beta > 1 - \frac{H}{p}$; and $Q^*_{I1} = 2N\overline{F}(p)$, if $\beta \le 1 - \frac{H}{p}$. The following theorem compares the optimal order quantities for when the retailer ignores consumer panic buying and when it does not.

THEOREM 4. *(a) Suppose* $\gamma = 1$.

(i) $Q^*_1 = Q^*_{I1}$, *if* $(\beta, \overline{v}) \in \Omega^n_2$ *and* $(\beta, \overline{v}) \in \{(\beta, \overline{v}) \in \Omega^n_1 | \overline{v} < p + \frac{h}{1-\beta}\}$;

(ii) $Q^*_1 \ne Q^*_{I1}$, *if* $(\beta, \overline{v}) \in \{(\beta, \overline{v}) \in \Omega^n_1 | \overline{v} \ge p + \frac{h}{1-\beta}\}$. *Furthermore,* $|Q^*_1 - Q^*_{I1}|$ *is increasing in* β *when* $\beta \in (0, 1 - \frac{H}{p})$, *and decreasing in* β *when* $\beta \in (1 - \frac{H}{p}, 1 - \frac{h}{\overline{v}-p})$.

(b) Suppose $0 < \gamma < 1$.

(i) $Q^*_1 = Q^*_{I1}$, *if* $(\beta, \overline{v}) \in \Omega^v_2$ *and* $(\beta, \overline{v}) \in \{(\beta, \overline{v}) \in \Omega^v_1 | \overline{v} < p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\}$;

(ii) $Q^*_1 \ne Q^*_{I1}$, *if* $(\beta, \overline{v}) \in \Omega^v_3$ *and* $(\beta, \overline{v}) \in \{(\beta, \overline{v}) \in \Omega^v_1 | \overline{v} \ge p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\}$. *Furthermore,* $|Q^*_1 - Q^*_{I1}|$ *is increasing in* β *when* $\beta \in (\beta_t, 1 - \frac{H}{p})$, *and decreasing in* β *when* $\beta \in (1 - \frac{H}{p}, (1 - \frac{h}{\overline{v}-p})^\gamma)$.

Theorem 4 shows that the retailer can ignore consumer panic buying behavior when the supply reliability is high or the consumer desirability for the product is low. Otherwise, the retailer should take consumer behavior into account. As in Figs. 1 (or 2) when $(\beta, \overline{v})$ lies in the area below the curve $\max\{p(H+h)/H, p + h/(1-\beta)\}$ (or, $\max\{v_t(\beta), p + h/(1-\beta^{\frac{1}{\gamma}})\}$), the retailer can ignore consumer behavior. When $(\beta, \overline{v})$ lies in the area above this curve, the retailer should acknowledge consumer behaviors. Theorem 4 also shows that the deviation between ignoring consumer behaviors and acknowledging them is the most significant when supply reliability is close to $1-\frac{H}{p}$. Hence, we define a critical ratio, $\hat{\beta}$, which satisfies

$$\hat{\beta}=1-\frac{H}{p}.$$

Fig. 3 illustrates these results for parameters $p=5$, $H=1$, $h=0.5$ $N=100$ and $\overline{v}=10$.

We now consider the loss in profit from ignoring consumer behavior, and we consider how this loss changes with model parameters. Define the percentage of profit loss (PL) caused by ignoring consumer behavior as

$$PL=\frac{\Pi^*-\Pi_I^*}{\Pi^*}, \tag{32}$$

where Π^* is the retailer's optimal profit when considering consumer behavior and Π_I^* is the retailer's profit when it ignores consumer behavior (but consumer behavior still actually exists, i.e., the corresponding profit when choosing Q_{I1}^*). When the retailer's order quantity is less than the demand in Period 1, there will be lost sales. In this case, the consumer arrival sequence affects the retailer's expected profit. We assume a uniform arrival sequence for the consumers (refer to the next section for the details of this assumption). When stockout occurs in Period 1, the retailer's expected profit is calculated based on a uniform arrival sequence.

We can show that the cost of ignoring consumer behavior is the highest when the supply reliability approaches the critical ratio $\hat{\beta}$. The following numerical examples demonstrate how PL changes with supply reliability, consumers' desirability for the product, and the degree of consumer risk aversion. The parameter values $p=5$, $H=1$, and $h=0.5$ are used in the following examples.

Table 2. The percentage of profit loss caused by ignoring consumer behavior versus supply reliability and consumer desirability for the product; $\gamma=0.5$.

$\overline{v}$ \ PL \ β	0.15	0.35	0.55	0.75	0.85	0.9	0.95	1
6	0	0	0	0	0	0	0	0
8	3.8%	4.2%	5%	5.5%	15.8%	5.8%	0	0
10	6.4%	6.6%	7.1%	7.4%	22%	17.3%	0	0
12	7.4%	7.6%	7.9%	8.1%	24.1%	20.8%	10.9%	0
14	8%	8.1%	8.4%	8.5%	25.3%	22.5%	15.6%	0

Table 3. The percentage of profit loss caused by ignoring consumer behavior versus degree of consumer risk aversion; $\overline{v}=10$.

β \ PL \ γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.75	8.8%	8.5%	8.1%	7.8%	7.4%	6.9%	6.5%	6.1%	5.7%
0.85	26.7%	0.256	24.5%	23.3%	22%	20.6%	19.1%	17.4%	15.7%

When supply reliability is either high or low, the percentage of profit loss caused by ignoring consumer behavior is relatively low. Nevertheless, when supply reliability is moderate (around $\hat{\beta}$), the percentage of profit loss can be very high (it can be as large as 25% as shown in Table 2). We interpret this observation as follows. When supply reliability is high, consumer panic buying behavior is not significant and ignoring consumer behavior will not incur much loss. When supply reliability is low, many consumers tend to purchase two units of product in Period 1. If the retailer accounts for consumer behavior, the retailer will place a large order for Period 1 to satisfy the demand, or the retailer will put a lot of product into inventory to hedge against the risk of supply disruption in Period 2. Coincidentally, if the retailer ignores consumer behavior then it is also optimal for the retailer to make an order before Period 1 to satisfy demand in both periods. Therefore, when supply reliability is low, the cost of ignoring consumer behavior is not very high. When supply reliability is moderate, ignoring consumer behavior makes the retailer incur higher holding costs (if $\beta<\hat{\beta}$) or higher stockout costs ($\beta>\hat{\beta}$). As we can see, the stockout cost p is much higher than the holding cost H. It follows that the percentage of profit loss is much higher when β approaches $\hat{\beta}$ from above than from below.

The percentage of profit loss caused by ignoring consumer behavior is increasing with respect to consumer desirability for the product. When consumer desirability for the product increases, consumer behavior becomes more significant to the retailer. Thus, the percentage of profit loss caused by ignoring consumer behavior is larger in this case.

Finally, the percentage of profit loss caused by ignoring consumer behavior is higher when the degree of consumer risk aversion is higher.

7. Capacity constraint and Quota Policy

In this section, we consider the case where the retailer have limited capacity and examine the resultant changes to the retailer's inventory decision. More importantly, we study the mechanism of quota policy (i.e., limiting the maximum number of units that each consumer can purchase) and evaluate its effectiveness for dealing with consumer panic buying behavior under limited-capacity case.

There are various practical reasons to consider a capacity constraint, e.g., the vendor has limited product supply or the retailer has limited storage space. When there is supply shortage and consumer panic buying behavior, quota policy has been often observed in practice. For example, during the milk powder crisis in Hong Kong, each consumer from mainland China can only purchase a maximum of two tins of milk powder; he/she would be heavily penalized if carrying more than two tins through the border between Hong Kong and mainland.

7.1 Impact of Capacity Constraint on Retailer's Inventory Decision

We suppose that the retailer's order cannot exceed K units during each period. To focus on the non-trivial cases, we assume that the capacity K is greater than or equal to the one-period demand without consumer panic buying, i.e., $K \geq N\overline{F}(p)$.

First consider the case where $K \geq N\Big(\overline{F}(p) + \overline{F}(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}})\Big)$. In this case, the retailer can satisfy all consumer demand in Period 1. The retailer must decide whether or not to carry inventory to increase the fulfillment rate in Period 2 and how much inventory to carry. We find that the managerial insights here are similar to those in Section 5, so we omit the discussion here.

Next, consider the case where $N\overline{F}(p) \leq K < N\Big(\overline{F}(p) + \overline{F}(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}})\Big)$. The retailer cannot meet all consumer demand in Period 1 so the retailer has lost sales in Period 1. We assume that consumers who were not able to purchase in Period 1 will still come to the retailer to purchase in Period 2. From the retailer's point of view, there are two types of consumers: consumers purchasing one unit of product in Period 1 (we refer to these consumers as Type-1 consumers), and consumers purchasing two units of product in Period 1 (Type-2 consumers). Under Constraint (33), the retailer cannot serve all consumers in Period 1. Although the retailer always sells K units of product in Period 1, the arrival sequence of consumers does affect the demand in Period 2. If all the Type-2 consumers arrive before the Type-1 consumers, then the demand in Period 2 will be lower than it would be if all the Type-1 consumers arrived before the Type-2 consumers.

To simplify our exposition, denote $K = \delta N\overline{F}(p)$. Hereafter we focus on the case where

$$1 \leq \delta < 1 + \frac{\overline{F}(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}})}{\overline{F}(p)}. \tag{33}$$

We assume a uniform arrival distribution for consumers as follows. Suppose the proportion of Type-2 consumers among the whole consumer population is ξ and so far x consumers have arrived. Among those x consumers, ξx are Type-2 consumers and $(1-\xi)x$ are Type-1. Under a uniform arrival pattern, for a large population of consumers (i.e., large N), the number of Type-2 consumers served in Period 1 is

$$\frac{K}{N\overline{F}(p) + N\overline{F}\Big(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\Big)} \cdot N\overline{F}(p) \cdot \frac{\overline{F}\Big(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\Big)}{\overline{F}(p)} = K\frac{\overline{F}\Big(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\Big)}{\overline{F}(p) + \overline{F}\Big(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\Big)}. \tag{34}$$

The first term in the left-hand-side (LHS) of Eqn. (34) is the total number of available units of product over the total Period 1 demand (the percentage of total demand served). The second term in the LHS of Eqn. (34) represents the total number of consumers, including Type-1 and Type-2 consumers. The product of the first two terms represents the total number of served consumers. Here, we use the percentage of served demand to estimate the percentage of served consumers based on a uniform arrival pattern and a large population of consumers. The third term in the LHS of Eqn. (34) is the percentage of the Type-2 consumer among all consumers. Denote

$$\theta = \frac{\overline{F}\left(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\right)}{\overline{F}(p) + \overline{F}\left(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\right)}. \tag{35}$$

Recall that consumer valuation is uniformly distributed on $[0, \overline{v}]$. Then $\theta = \frac{(1-\beta^{\frac{1}{\gamma}})(\overline{v}-p)-h}{2(1-\beta^{\frac{1}{\gamma}})(\overline{v}-p)-h}$ if $p + \frac{h}{1-\beta^{\frac{1}{\gamma}}} \leq \overline{v}$; and $\theta = 0$ if $p + \frac{h}{1-\beta^{\frac{1}{\gamma}}} > \overline{v}$. θ indexes the percentage of Type-2 consumers served in Period 1.

LEMMA 1. *θ is increasing with respect to $\overline{v}$ and decreasing with respect to h, β, γ, and p.*

Proof. If $p + \frac{h}{1-\beta^{\frac{1}{\gamma}}} > \overline{v}$, the result follows immediately. Next we consider the case where $p + \frac{h}{1-\beta^{\frac{1}{\gamma}}} \leq \overline{v}$. By a simple calculation, we have

$$\theta = 1 - \frac{\overline{v} - p}{2(\overline{v} - p) - X} = \frac{1}{2} - \frac{X}{4(\overline{v} - p) - 2X}, \tag{36}$$

where $X = h/(1 - \beta^{\frac{1}{\gamma}})$. Clearly, X is increasing in h, β, and γ and θ is decreasing in X. Thus, θ is decreasing in h, β, and γ. By the second equation of (36), we conclude that θ is increasing in $\overline{v}$ and decreasing in p. ■

Lemma 1 shows that the number of Type-2 consumers served in Period 1 becomes larger when consumer product desirability increases, or when consumer holding cost, supply reliability, degree of consumer risk aversion, or price decreases. Given lower holding costs, supply reliability, or price, or given a higher product desirability or degree of risk aversion, more consumers tend to purchase 2 units of product in Period 1.

Under the capacity constraint (33), the retailer will order $Q_1^* = K$ and $Q_2^* = 2N\overline{F}(p) - Q_1^*$. In this case, the retailers expected profit is

$$\Pi^* = pK + \beta p(N\overline{F}(p) - K\theta), \tag{37}$$

where pK represents the Period 1 profit and $\beta p(N\overline{F}(p) - K\theta)$ represents the expected Period 2 profit. Obviously, a larger value of θ corresponds to lower expected retailer profit. By Lemma 1, we know that the retailer will have a higher profit when h, β, or γ are larger.

7.2 Effectiveness of Quota Policy

We have shown that the retailer's profit may suffer under the capacity constraint (33). Some portion of Period 2 demand is switched to Period 1, which leads to unsatisfied demand in Period 1 because of supply shortage. For this reason, retailers in practice often impose fixed quota policies on consumers and allow each consumer to buy only one unit of product at a time. We seek to determine when the retailer should use a fixed quota policy.

Suppose that the retailer implements a fixed quota policy. If the retailer orders Q_{f1} units of product at the beginning of Period 1, then it is optimal for the retailer to order $Q_{f2} = 2N\overline{F}(p) - Q_{f1}$ at the beginning of Period 2. The retailer's expected profit can be written as a function of Q_{f1} (the subscript f stands for fixed quota):

$$\begin{aligned}\Pi_f(Q_{f1}) &= pN\overline{F}(p) - H(Q_{f1} - N\overline{F}(p)) + \beta pN\overline{F}(p) + (1-\beta)p(Q_{f1} - N\overline{F}(p)) \\ &= (\beta+1)pN\overline{F}(p) + [(1-\beta)p - H](Q_{f1} - N\overline{F}(p)).\end{aligned} \tag{38}$$

The retailer should order $Q_{f1} \in [N\overline{F}(p), K]$ to maximize its expected profit in (38). If $\beta > 1 - H/p$, then the optimal order quantity is $Q^*_{f1} = N\overline{F}(p)$ and the optimal expected profit is $\Pi^*_f = \Pi_f(Q^*_{f1}) = (1+\beta)pN\overline{F}(p)$. If $\beta \le 1 - H/p$, then the optimal order quantity is $Q^*_{f1} = K$ and the optimal expected profit is $\Pi^*_f = \Pi_f(Q^*_{f1}) = (1+\beta)pN\overline{F}(p) + [(1-\beta)p - H](K - N\overline{F}(p))$. The next theorem identifies a threshold for the retailer regarding whether or not to implement a fixed quota policy.

THEOREM 5. *There exists a threshold $\delta_t > 1$ such that if $\delta \le \delta_t$, then the retailer should implement a fixed quota policy, and if $\delta > \delta_t$, then the retailer should not, where*

$$\delta_t = \begin{cases} \frac{1}{1-\beta\theta}, & if\ \beta > 1 - \frac{H}{p}; \\ \frac{H+\beta p}{\beta p(1-\theta)+H}, & if\ \beta \le 1 - \frac{H}{p}. \end{cases} \tag{39}$$

Theorem 5 produces a threshold such that when the capacity K is smaller than the threshold, the retailer should implement a fixed quota policy. When the capacity K is larger than the threshold, the retailer should not implement a fixed quota policy.

PROPOSITION 4. *δ_t is increasing with respect to $\overline{v}$, and decreasing with respect to γ, h, and H.*

Proposition 4 indicates that the fixed quota policy is more beneficial to the retailer when consumer desirability for the product is high, the consumer holding cost is low, the consumer degree of risk aversion is high, or the retailer's holding cost is low. When consumer desirability for the product increases, consumer holding costs decrease, or the degree of consumer degree of risk aversion increases, more consumers purchase two units of product in Period 1. Consequently, there are more lost sales in Period 1 and there is less demand in Period 2. A fixed quota policy is attractive in this situation. When the retailer's holding cost decreases, the retailer has a greater incentive to carry inventory if supply reliability is low. Thus, it is more attractive for the retailer to implement a fixed quota policy to avoid lost sales in Period 1.

Next we use two numerical examples to study how the attractiveness of a fixed quota policy changes with supply reliability and selling price. The first example (Fig. 3) is based on parameters: $H = 2$, $h = 0.5$, $\overline{v} = 10$, and $\gamma = 0.5$, and it characterizes the relationship between δ_t and β (or p). The second example (Fig. 4) is based on parameters: $H = 2$, $h = 0.5$, $\overline{v} = 10$, $\gamma = 0.5$, $N = 1000$, and $\delta = 1.3$, and it characterizes the relationship between $\Pi_f^* - \Pi^*$ and β (or p). Figures 3 and 4 provide the following observations:

First, the attractiveness of a fixed quota policy is initially increasing and then decreasing in the level of supply reliability. In addition, both δ_t and $\Pi_f^* - \Pi^*$ reach their maxima at $\beta = 1 - H/p$. These results are somewhat counter-intuitive since one may think that lower supply reliability would induce more Type-2 consumers and make a fixed quota policy more attractive. When $\beta \leq 1 - H/p$, the retailer will order $Q_{f1}^* = K$ and increase its profit by $p[N\overline{F}(p) - (1-\theta)K]$ under a fixed quota policy. However, implementing a fixed quota policy also incurs some costs: (i) Under the fixed quota policy, the retailer carries extra inventory at the end of Period 1 at cost $H(K - N\overline{F}(p))$. (ii) To cut lost sales, the fixed quota policy postpones $K\theta$ consumer requests for the second unit of product to Period 2. The postponement exposes $[N\overline{F}(p) - (1-\theta)K]$ consumer requests to the risk of supply disruption (note that the retailer has $K - N\overline{F}(p)$ units of inventory at the beginning of Period 2). The retailer's expected profit decreases by $(1-\beta)p[N\overline{F}(p) - (1-\theta)K]$. In summary, the net benefit of a fixed quota policy is $\beta p[N\overline{F}(p) - (1-\theta)K] - H(K - N\overline{F}(p))$. Note that the holding cost $H(K - N\overline{F}(p))$ does not depend on β, so we only consider $\beta p[N\overline{F}(p) - (1-\theta)K]$. As supply reliability increases, although the quantity of lost sales $N\overline{F}(p) - (1-\theta)K$ decreases, β increases more than the quantity of lost sales decreases. Thus, a fixed quota policy is more attractive to the retailer as supply reliability increases when $\beta \leq 1 - H/p$.

When $\beta > 1 - H/p$, it is optimal for the retailer to order only $N\overline{F}(p)$ before Period 1. On the one hand, the fixed quota policy increases the retailer's profit by $p[N\overline{F}(p) - (1-\theta)K]$ by cutting lost sales. On the other hand, implementing the fixed quota policy postpones $K\theta$ consumer requests for the second unit of product to Period 2 and exposes them to the risk of supply disruption. Thus, the fixed quota policy lowers the retailer's expected profit by $(1-\beta)K\theta$. In this case, the net benefit of the fixed quota policy is $p[N\overline{F}(p) - (1-\theta)K] - (1-\beta)pK\theta = \beta p[N\overline{F}(p) - (1-\theta)K] - (1-\beta)p[K - N\overline{F}(p)]$. When β is not too large, the quantity of lost sales $N\overline{F}(p) - (1-\theta)K$ decreases less significantly than β increases and the cost $(1-\beta)p[K - N\overline{F}(p)]$ decreases. Thus, $\beta p[N\overline{F}(p) - (1-\theta)K] - (1-\beta)p[K - N\overline{F}(p)]$ increases and the fixed quota policy is increasingly favorable as supply reliability increases. When β approaches 1, the quantity of lost sales $(1-\beta)p[K - N\overline{F}(p)]$ decreases dramatically. These effects dominate the effects caused by increasing β and $-(1-\beta)p[K - N\overline{F}(p)]$.

Figure 3 **The threshold δ_t v.s. supply reliability and price**

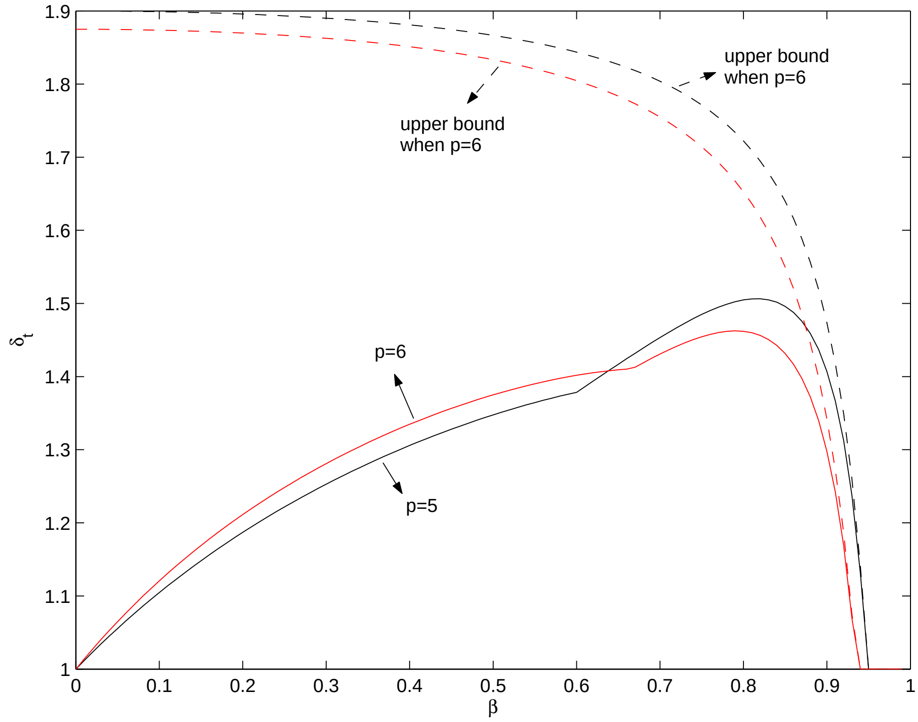
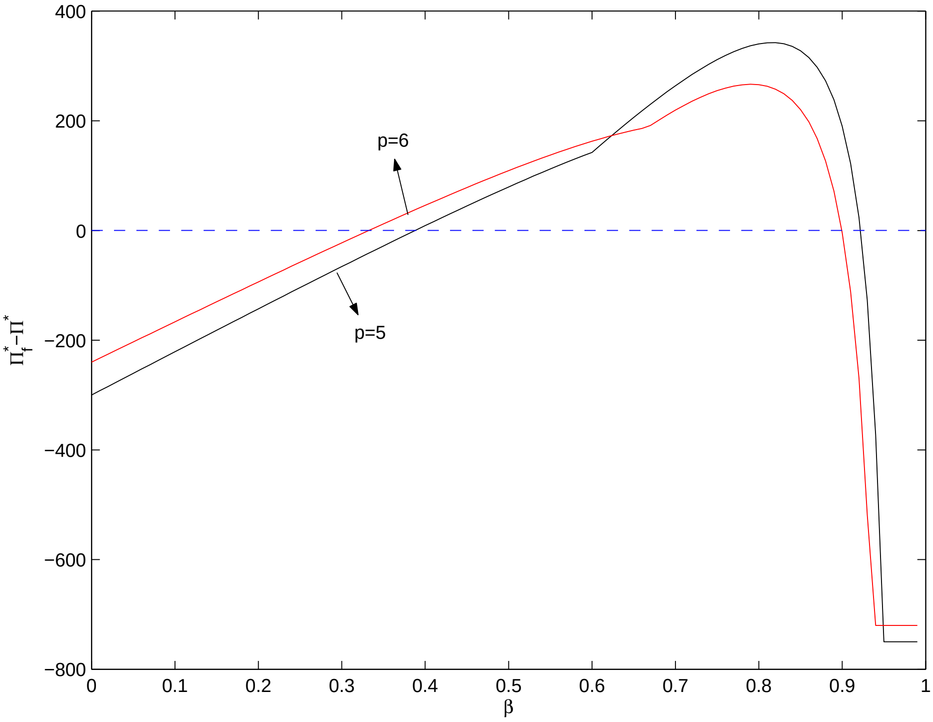


Figure 4 **The net benefit of fixed quota policy v.s. supply reliability and price**



Therefore, the fixed quota policy becomes less attractive when β is sufficiently large and approaches 1.

In the case $\beta \leq 1 - H/p$, the net benefit is $\beta p[N\overline{F}(p) - (1-\theta)K] - H(K - N\overline{F}(p))$, which is increasing in β as β approaches $1 - H/p$. In the case $\beta > 1 - H/p$, the net benefit is $\beta p[N\overline{F}(p) - (1-\theta)K] - (1-\beta)p[K - N\overline{F}(p)]$, which is also increasing in β if β is close to $1 - H/p$.

We observe that the fixed quota policy is more attractive as the selling price increases when supply reliability is low, but the opposite is true when supply reliability is high. Increasing the selling price has two effects: (a) decreasing the total number of consumers who purchase and decreasing the number of Type-2 consumers (both of which decrease the lost sales in Period 1); (b) increasing the retailer's profit margin. When supply reliability is low, the net benefit of the fixed quota policy is $\beta p[N\overline{F}(p) - (1-\theta)K] - H(K - N\overline{F}(p))$, and in this case effect (a) is dominated by effect (b). Thus, the fixed quota policy is increasingly favorable as the selling price increases when the supply reliability is low. When the supply reliability is high, the net benefit of the fixed quota policy is $\beta p[N\overline{F}(p) - (1-\theta)K] - (1-\beta)p[K - N\overline{F}(p)]$. In this case, as the selling price increases, both terms of the above expression increase and counteract to each other. Hence, effect (b) becomes less significant and is dominated by effect (a). The attractiveness of the fixed quota policy decreases as the selling price increases when supply reliability is high.

8. Conclusion

We study consumer panic buying under supply disruptions and investigate how the retailer should adapt inventory and quota policies to deal with to both demand-side and supply-side challenges.

First, we analyze the main drivers for consumer panic buying. We show that a consumer will stockpile product for future consumption only if his valuation of the product is above a threshold. Moreover, we find that consumers are more likely to stockpile when the price of the product or the consumer holding cost is low, or when the consumers are risk averse, or less certain about obtaining the product in the next period.

Second, we derive the retailer's optimal inventory policy and evaluate the effectiveness of quota policy for the retailer with limited capacity. We show that when consumers are risk neutral, the retailer should carry enough safety inventory to cover all demands in future period if both the supplier reliability and consumers' desirability of the product is lower than certain threshold values; otherwise, he should carry no safety inventory at all. On the other hand, when consumers are risk averse, keeping safety inventory to satisfy only *partial* of the demand is optimal under some conditions. Furthermore, we show that only when the retailer's capacity is below a certain level implementing quota policy is beneficial to the retailer. Quota policy is more helpful when consumer desirability for the product is higher, the consumer holding costs are lower, the retailer's holding cost is lower, or the degree of consumer risk aversion is higher.

Finally, we demonstrate the substantial potential loss to the retailer if it ignores consumer behavior changes. We identify a critical ratio and show that the cost of ignoring consumer behavior is the most significant when supply reliability level is close to the ratio.

Appendix

Proof for Proposition 1.

Proof. We only prove that $T(p,h,\alpha_c)$ is increasing in p, since the proofs for h and α_c are similar. Since $U(x)$ is an increasing concave function, $U(v-p-h)/U(v-p)$ is increasing in v and decreasing in p. For any $p_1 < p_2$, we have

$$\frac{U(T(p_1,h,\alpha_c)-p_2-h)}{U(T(p_1,h,\alpha_c)-p_2)} < \frac{U(T(p_1,h,\alpha_c)-p_1-h)}{U(T(p_1,h,\alpha_c)-p_1)} = \alpha_c = \frac{U(T(p_2,h,\alpha_c)-p_2-h)}{U(T(p_2,h,\alpha_c)-p_2)},$$

which indicates that $T(p_1,h,\alpha_c) < T(p_2,h,\alpha_c)$. ■

Proof for Proposition 2.

Proof. Recall the definition of T in Eqn. (5). Both $U(v-p-h)/U(v-p)$ and $W(v-p-h)/W(v-p)$ are increasing in v, so we only need to prove that

$$\frac{U(v-p-h)}{U(v-p)} \le \frac{W(v-p-h)}{W(v-p)} \tag{40}$$

for any $v \ge p+h$. Clearly, to prove Inequality (40), we only need to prove that, for any x and y:

$$\frac{U(x-y)}{W(x-y)} \le \frac{U(x)}{W(x)}. \tag{41}$$

Let $k(x) = U(x)/W(x)$. Then we have

$$k^{'}(x) = \frac{U^{'}(x)W(x) - W^{'}(x)U(x)}{[W(x)]^2} = \frac{U^{'}(x)[\varphi(U(x)) - \varphi^{'}(U(x))U(x)]}{[W(x)]^2}.$$

Since φ is an increasing concave function with $\varphi(0) = 0$,

$$\varphi(U(x)) + \varphi^{'}(U(x))[0 - U(x)] \ge \varphi(0) = 0,$$

which indicates that $k(x)$ is an increasing function. Thus, Inequality (41) holds. ■

Proof for Proposition 3.

Proof. Part (i). We only provide a proof with respect to γ, since the other cases are similar. Let

$$Y(\beta,\gamma) = \gamma\beta^{1-\frac{1}{\gamma}} + (1-\gamma)\beta - 1.$$

Differentiating $Y(\beta,\gamma)$ with respect to γ gives

$$\frac{\partial Y}{\partial \gamma}(\beta,\gamma) = \beta^{1-\frac{1}{\gamma}}(1 + \frac{1}{\gamma} ln\beta) - \beta.$$

Since $\frac{\partial Y}{\partial \gamma}(1,\gamma)=0$ and

$$\frac{\partial^2 Y}{\partial\gamma\partial\beta}(\beta,\gamma)=(1+\frac{1}{\gamma}(1-\frac{1}{\gamma})ln\beta)\beta^{-\frac{1}{\gamma}}-1>0,$$

for any $0\le\beta\le 1$ we have $\frac{\partial Y}{\partial \gamma}(\beta,\gamma)\le 0$, which implies that $\frac{\partial X}{\partial \gamma}(\beta,\gamma,p,H)\le 0$, where $X(\beta,\gamma,p,H)$ is defined in (30). For any $0<\gamma_1<\gamma_2\le 1$, we have

$$X(\beta_t(\gamma_1,p,H),\gamma_2,p,H)\le X(\beta_t(\gamma_1,p,H),\gamma_1,p,H)=0,$$

which, together with the fact that $X(\beta,\gamma,p,H)$ is decreasing with respect to β, implies that $\beta_t(\gamma_1,p,H)\ge\beta_t(\gamma_2,p,H)$.

Part (ii). Π_2' in (25) is decreasing in β and H. Thus, by the concavity of Π_2 with respect to T, we have that T^o is decreasing in β and H. To prove that T^o is increasing with respect to p, we define $\widetilde{T}=T-p$. Then we have

$$g(\widetilde{T})\equiv\Pi_2'(T)=-\frac{N}{\overline{v}}\{H-(p-\frac{H}{1-\beta})[(1-\frac{h}{\widetilde{T}})^{\gamma-1}(1-\frac{(1-\gamma)h}{\widetilde{T}})-1]\}. \qquad (42)$$

$g(\widetilde{T})$ is decreasing in T and increasing in p. Hence, $\widetilde{T}^o$ such that $g(\widetilde{T}^o)=0$ is increasing in p, which indicates that $T^o=\widetilde{T}^o+p$ is also increasing with respect to p. ■

Proof for Theorem 4.

Proof. We only provide a proof of Part (a) since the proof of Part (b) is similar. If $(\beta,\overline{v})\in\Omega_2^n$, both Q^* and Q_I^* are equal to $2N\overline{F}(p)$. If $(\beta,\overline{v})\in\{(\beta,\overline{v})\in\Omega_1^n|\overline{v}<p+\frac{h}{1-\beta}\}$, then $Q^*=Q_I^*=N\overline{F}(p)$. For other cases, $Q^*\neq Q_I^*$. If $Q^*\neq Q_I^*$ and $\beta\in(0,1-\frac{H}{p})$, $|Q^*-Q_I^*|=Q_I^*-Q^*=N[\overline{F}(p)-\overline{F}(p+\frac{h}{1-\beta^{\frac{1}{\gamma}}})]$, which is increasing with respect to β. If $Q^*\neq Q_I^*$ and $\beta\in(1-\frac{H}{p},1)$, then $|Q^*-Q_I^*|=Q^*-Q_I^*=N\overline{F}(p+\frac{h}{1-\beta^{\frac{1}{\gamma}}})$, which is decreasing with respect to β. ■

Proof for Theorem 5.

Proof. Note that the retailer will implement fixed quota policy if and only if $\Pi_f^*\ge\Pi^*$. If $\beta>1-H/p$, then $\Pi^*-\Pi_f^*=pN\overline{F}(p)[(1-\beta\theta)\delta-1]$. If $\beta\le 1-H/p$, then $\Pi^*-\Pi_f^*=N\overline{F}(p)\{[\beta p(1-\theta)+H]\delta-(H+\beta p)\}$. By the equations above, one can easily come to the results. ■

Proof for Proposition 4.

Proof. From Eqn. (39), we know that δ_t is increasing in θ. Thus, δ_t is increasing in $\overline{v}$ and decreasing in γ and h by Lemma 1. By Eqn. (39), when $\beta\le 1-H/p$, $\delta_t=1+\frac{\theta\beta p}{\beta p(1-\theta)+H}$, which is decreasing in H. When $\beta>1-H/p$, δ_t is independent of H. Hence, δ_t is nonincreasing with respect to H. ■

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