Submitted to *Management Science*
manuscript

Consumer Panic Buying and Fixed Quota Policy

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We study consumer panic buying under supply disruption and investigate how the retailer should respond to panic buying through inventory, pricing and fixed quota policy. First, we derive the threshold of consumer's product valuation above which a consumer will stockpile products for future consumption. Then we conduct rational expectation equilibrium analysis to charaterize the retailer's inventory policy and the cost of ignoring consumer behaviors. Interestingly, we find that cost is the most significant when supply reliability is intermediate. Furthermore, we examine the effectiveness of retailer's fixed quota policy under capacity constraint. We derive the threshold value of the retailer's capacity below which it is beneficial for the retailer to implement fixed quota policy. We also show that the fixed quota policy is more attractive to the retailer when consumers' desirability for the product is higher, the consumers' holding cost is lower, the retailer's holding cost is lower, or the consumers' degree of risk aversion is higher. Finally, we consider two extensions to our basic model, namely responsive pricing and random yield models, and show that our key managerial insights are robust.

Key words: Panic buying; strategic consumer behavior; supply disruption; fixed quota
History: submitted in May 2011

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1. Introduction

Consumer panic buying, also known as consumer hoarding, refers to the act of people buying unusually large amounts of product to avoid future shortage. Panic buying has been frequently observed recently. For example, when Hurricane Katrina disabled most of the oil drilling facilities in the U.S. Gulf coast region in 2005, consumer hoarding behavior and long lines were observed at gasoline stations (Wall Street Journal, 2005). When rice production in Australia reduced by 98% due to long period of drought in 2008, fears of insufficient rice supplies spread around the globe. Many regions, such as Vietnam, India and Hong Kong, saw consumers rushing to stores and exhausting rice supply (New York Times, 2008). Amid the latest nuclear crisis in Japan, worried shoppers stripped stores of salt in Beijing, Shanghai, San Fransico and other cities. (China Daily, 2011).

Consumer panic buying often leads to long waiting-lines, large-scale stock-out, great amount of anxiety, and thus has a significant negative impact on the market. It is important to fully understand the consumer panic buying behavior and how to mitigate it.

One of the primary reasons for consumer panic buying is the disruption of normal product supply. Supply disruption can occur due to various reasons, such as natural disasters, labor strikes, terrorist attacks, and government regulations. The relationship between supply disruption and consumer panic buying is quite complex in several aspects: (1) Supply disruption may lead to consumer panic buying. Consumer panic buying can be viewed as a way of inventory accumulation to hedge supply disruption. (2) Consumer panic buying may further exaggerate the consequences of supply disruption, as an abnormally high demand leads to substantial stock-out and induces more panic buying. (3) Retailer decisions such as increasing price, limiting supply, and posing purchasing quota may greatly affect consumer behaviors. Although these practices normally help to repress demand, they may also increase consumers' anxiety about supply shortage and make panic buying worse.

Due to the above tight coupling and complex relationships, it is natural to jointly study the impact of consumer panic buying and the proper retailer decisions under supply disruption. Unfortunately, the related studies of these two aspects in the literature are largely disjoint. Consumer panic buying has been mostly studied by economists, psychologists, and sociologists, with a common focus on qualitative studies based on empirical data. The impact of supply disruption has been mostly studied in the supply chain management literature, which mostly ignores strategic consumer responses.

In this paper, we aim at addressing the gap. Section 2 reviews the related literature. Section 3 presents the model. In Section 4, we characterize consumer behaviors. In Section 5, we conduct equilibrium analysis to study the retailer's inventory policy and the cost of ignoring consumer behaviors. In Section 6, we consider the effectiveness of fixed quota policy under the capacity constraint of the retailer. Section 7 provides some extensions to the basic model, in which the first considers retailer's responsive pricing under supply disruption and the second extend the supply disruption case to the random proportional yield one. Finally, we conclude the paper in Section 8.

2. Literature review

Our work is related to the existing literature on strategic consumer behavior (e.g., Aviv and Pazgal (2008), Liu and van Ryzin (2010), van Ryzin and Liu (2008), Shen and Su (2007), Su and Zhang (2008), Su (2010), and Allon and Bassamboo (2011)). Shen and Su (2007) provided a good review of customer behavior models in the revenue management and auction literatures. Aviv and Pazgal (2008) analyzed a model with a Poisson stream of strategic customers and a single price reduction point at T. The authors numerically demonstrated that neglecting strategic customer behavior can lead to substantial loss. Liu and van Ryzin (2010) studied the effects of capacity rationing, and found that competition may hinder the profitability of capacity rationing. Su (2010) used rational expectations to study consumer stockpiling under dynamic pricing setting. The major difference between our paper and the above literature is that we focus on studying the consumer stockpiling behavior (panic buying) under supply disruption, when consumer primarily wants to avoid future supply shortage instead of taking advantages of price promotions.

Another related research stream is supply uncertainty. There are three approaches to model supply uncertainty: random lead-time, random yield, and supply disruption. The random lead-time model assumes that lead-time is a random variable. See Song and Zipkin (1996) for an excellent literature review. The random yield model assumes that the realized supply level is a random function of the order/capacity level. Examples are Yano and Lee (1995), Gerchak and Parlar (1990), Parlar and Wang (1993), Swaminathan and Shanthikumar (1999), Federgruen and Yang (2008), Wang et al. (2008), Kazaz (2008, 2004), Deo and Corbett (2008). For instance, Tang and Yin (2007) developed a two-stage stochastic model to analyze the impact of responsive pricing under supply uncertainty, i.e., the retailer would specify the order quantity first and then decide on the retail price after observing the realized supply yield. The authors showed that the responsive pricing always lead to a higher expected profit for the retailer. Shou et al. (2009) examined the coordination incentives of two competing supply chains which are subject to supply uncertainty. They showed that supply chain coordination is a dominant strategy. Nevertheless, if supply risk is low, coordination may decrease supply chain profit which results in a prisoner's dilemma. Supply disruption models the uncertainty of supply as one of two states: "up" or "down". The orders are fulfilled on time and in full when the supplier is "up", and no order can be fulfilled when the supplier is "down". Snyder et al. (2010) provides a comprehensive review of supply disruption papers. In our paper, we first use supply disruption models to derive key managerial insights. Later on we show that these results also hold under random yield model.

There are very few papers in the literature that explicitly considered the impact of supply uncertainty on customer demand. Rong et al. (2008) considered the one-shot game interaction between an unreliable supplier and multiple retailers. The retailers may inflate their order quantities in order to obtain their desired allocation from the supplier, a behavior known as the rationing game. It was shown that no Nash Equilibrium of the retailers' order quantities exists for a certain class of supply processes when the supplier applies the well known proportional allocation rule. Furthermore, if the retailers make a certain reasonable assumption about their competitors' behavior, both the bullwhip effect and reverse bullwhip effect occur. Rong et al. (2009) investigated three different pricing strategies during supply disruption, namely naive, one-period correction (1PC), and regression approaches. The authors showed that the naive pricing strategy is superior to either of the more sophisticated ones, both in terms of the firm's profit and the magnitude of the customer's order variability. However, the authors did not model customer stockpiling directly. This behavior was implicitly reflected by the difference between the order quantity and the underlying demand, which is assumed to be a linear function of the anticipation of price change.

The key differences between our paper and the above-mentioned papers are: 1) we develop a two-period rational expectation model to explicitly characterize consumers' panic buying behavior facing potential supply disruption threat; 2) Different from the previous papers' focus on retailers' rationing game and supplier's allocation policy, we focus on the impact of consumer panic buying on retailer's inventory policies, and the effectiveness of retailer's responsive pricing and fixed-quota policies, which are common mitigation strategies used in practice.

3. Model

Consider a situation that a monopolist retailer sells a staple product to a mass of consumers over two periods: Periods 1 and 2. At the beginning of Period 1, the retailer orders Q_1 units of products (a decision) for sale, and the leftover products at the end of Period 1 are carried over for sale in Period 2 and incur a holding cost (let H be the holding cost rate per unit per period). At the beginning of Period 2, the retailer has an opportunity to replenish its inventory. However, the replenishment at the beginning of Period 2 cannot be always successful due to supply disruption. We assume that the supply disruption occurs with a probability $1-\beta$, i.e., the retailer can successfully make a replenishment at the beginning of Period 2 with a probability β. (To focus on the impact of expectation of future supply uncertainty, here we only consider that supply disruption occurs in Period 2, not Period 1). We assume that the retail price p for the product is exogenous and constant over the two periods. The procurement cost for the retailer is normalized to 0. The products leftover at the end of Period 2 is salvaged at a value of 0.

There are a large population of (N) consumers in the market, who possess heterogenous valuations over the product. The consumer valuation v is drawn from a common distribution $F(v)$. Each consumer demands one unit of the product for consumption in each period (the utility increase for consuming extra products is 0). At the beginning of each period, the consumers have an opportunity to purchase from the retailer. However, due to supply disruption, consumers may fail to get the products in Period 2, thus incur a utility loss of failing to consume the product in Period 2. To avoid this utility loss, a consumer may purchase 2 units in Period 1 (i.e., stockpile 1 unit) for consumption in Period 2. Nevertheless, at the end of Period 1, the unused product incurs a holding cost h per unit for the consumer.

The chronology of events is as follows:

1. At the beginning of Period 1, the retailer decides the first order quantity, Q_1, from its supplier. The order will be received in full by the retailer immediately.

2. Each consumer decides his purchase quantity $z_1 \in \{0,1,2\}$ in period 1.

3. During Period 1, $min\{1, z_1\}$ is consumed for each consumer. The leftover product, if any, incurs a holding cost.

4. At the beginning of Period 2, the retailer decides the second order quantity, Q_2, from its supplier. Due to supply disruption, the retailer may not receive the second inventory order.

5. Each consumer decides his purchase quantity $z_2 \in \{0,1\}$ in period 2. However, their demand may not be always fulfilled, depending on the availability of retailer's inventory. In particular, we assume the fulfill rate at the beginning of Period 2 is α, which is dependent on the probability of supply disruption and retailer's inventory decision. We further assume the all the consumer has rational expectations on this fulfill rate, which is an assumption commonly used in existing literature (e.g., Liu and Ryzin 2008, Su and Zhang 2009).

6. During Period 2, consumers consume the product on hand if he has any.

We assume the retailer is risk neutral, as in general a retailer has a large and well-diversified portfolio of products. We assume that the consumers are risk averse (the risk neutral case is considered as a special case). In particular, we assume a consumer with valuation v has a utility function $U_v(x) = U(x - r_v)$, where $U(\cdot)$ is an increasing concave function with $U(0) = 0$ (the risk neutral case is a special case with $U(x) = x$). We also assume that

$$\lim_{x \to +\infty} \frac{U(x-y)}{U(x)} = 1, \tag{1}$$

for any real number y. Many increasing concave utility functions satisfy Eqn. (1), such as $U(x) = 1 - exp(-rx)$ $(0 < r < +\infty)$, $U(x) = x^\gamma$ $(0 < \gamma \le 1)$, etc. In $U_v(\cdot)$, r_v is the reference point (please see Prospect Theory of Kahneman and Tversky 1979 for details). We assume that

$$r_v = \begin{cases} v - p, & if \quad v > p, \\ 0, & if \quad v \le p. \end{cases} \tag{2}$$

All the parameters and distribution functions are common knowledge to the retailer and consumers. To simplify our exposition, we let $\overline{F}(x) = 1 - F(x)$ and $x^+ = max\{x, 0\}$. The aim of this paper is to characterize the consumers' stockpiling behavior and the retailer's inventory policy with possibilities of supply disruption.

Table 1 provides a summary of the key notations in this paper.

4. Consumers' purchasing policy

In this section, we focus on consumers' stockpiling policy. To this end, we consider consumers' optimal purchasing policy in each period for given belief over the fulfill rate in Period 2. Note that based on rational expectation assumption, all the consumers' beliefs are the same and consistent

Table 1 Key notations

H (h)	Retailer's (consumers') holding cost rate per unit per period
p	Regular selling price of the product
N	Number of consumers
v	A consumer's valuation to the product
$F(v)$	Distribution of consumer valuation
$\overline{v}$	The highest valuation when $F(v)$ is assumed to be uniform distribution
β	Supply reliability
z_1 (z_2)	A consumer's purchasing quantity in Period 1 (Period 2)
Q_1 (Q_2)	The retailer's order quantity at the beginning of Period 1 (Period 2)
$U_v(\cdot)$	The utility function of a consumer with valuation v
r_v	The reference point of a consumer with valuation v
γ	A parameter measures consumers' degree of risk aversion
α (α_c)	The fulfill rate (consumers' belief on the fulfill rate) in Period 2, $0 \leq \alpha, \alpha_c \leq 1$
$u(\cdot)$	A consumer's expected utility
$\Pi(\cdot)$ Π^*	The retailer's (optimal) expected profit under the basic model
Q_{i1} (Q_{i2})	The retailer's order quantity at the beginning of Period 1 (Period 2) under Case i
$\Pi_i(\cdot)$ Π_i^*	The retailer's (optimal) expected profit under Case i $i = I$: the case of ignoring consumer behaviors; $i = f$: the case of fixed quota; $i = r$: the case of responsive pricing
PL	The retailer's profit loss of ignoring consumer behaviors
K	The retailer's capacity
δ	The retailer's capacity over demand of Period 1
p_r ($p_{r,c}$)	In responsive pricing model, (the consumers' belief over) the responsive price of Period 2
η (η_c)	In responsive pricing model, (the consumers' belief over) the probability of a consumer get what he purchases when supply disruption occurs
x	The yield proportion in the random proportional yield model
μ	The mean of x, $E(x) = \mu$
$B(x)$	The distribution of x

with the realized fulfill rate in equilibrium, hence we only need to consider the case that all consumers have homogenous belief over this fulfill rate. The case of heterogenous beliefs can also be analyzed, but the results in this section will not change. Let α_c be the consumers' belief over this fulfill rate and S be a random variable indicating whether a consumer gets what he purchases in Period 2. $S = 1$ means that the consumer gets what he purchases in Period 2 and $S = 0$ means he doesn't get. Consider a consumer with valuation v. Clearly, if $v \leq p$, then the consumer will not buy any products in Period 1 or 2. Here, we only consider the case $v > p$. In this case, the consumer's purchasing quantity at the beginning of Period 1, z_1, is either 1 or 2 and his purchasing quantity at the beginning of Period 2, z_2, equals $2 - z_1$. The consumer's payoff is

$$\pi = \begin{cases} 2(v-p) - h(z_1 - 1), & if \quad S = 1, \\ (v-p)z_1 - h(z_1 - 1), & if \quad S = 0. \end{cases} \tag{3}$$

In the case of $S = 1$, since the consumer gets what he purchases in both periods and $z_1 + z_2 = 2$, then the utility by consuming the products is always $2(v - p)$ regardless of z_1. In the case of $S = 0$, the consumer can only get what he purchases in Period 1, so the utility by consuming the products is $(v-p)z_1$. The term $h(z_1-1)$ refers to the consumer's holding cost. For given belief over the fulfill rate, α_c, the consumer's expected utility with z_1 and $z_2 = 2 - z_1$ is

$$u(z_1) = E_S U_v(\pi - r_v) = \alpha_c U\Big((v-p) - h(z_1-1)\Big) + (1-\alpha_c) U\Big((v-p-h)(z_1-1)\Big). \tag{4}$$

The following theorem identifies the optimal purchasing policy for the consumers.

THEOREM 1. *For any given belief over the fulfill rate in Period 2, α_c, there exists a threshold $T(p,h,\alpha_c) \geq p+h$ such that the optimal purchasing policy for the consumer with valuation v is as follows:*

1. *If $v \leq p$, then $z_1^*(\alpha_c) = z_2^*(\alpha_c) = 0$;*
2. *If $p < v \leq T(p,h,\alpha_c)$, then $z_1^*(\alpha_c) = z_2^*(\alpha_c) = 1$;*
3. *If $v > T(p,h,\alpha_c)$, then $z_1^*(\alpha_c) = 2,\ z_2^*(\alpha_c) = 0$.*

Proof. Case 1 is straightforward, so we focus on the proofs of Cases 2 and 3. By Eqn. (4), we have

$$u(1) = \alpha_c U(v-p),$$
$$u(2) = U(v-p-h).$$

Thus, $u(1) > (<) u(2)$ is equivalent to

$$\frac{U(v-p-h)}{U(v-p)} < (>) \alpha_c.$$

Let $g(v) = U(v-p-h)/U(v-p)$. It can be easily verified that $g(v)$ is increasing in v since $U(x)$ is an increasing concave function. In addition, we have $g(p+h) = 0$ and $lim_{v\to+\infty} g(v) = 1$ by Assumption (1). Therefore, there exists a number $T(p,h,\alpha_c) \geq p+h$ satisfying

$$\frac{U(T(p,h,\alpha_c)-p-h)}{U(T(p,h,\alpha_c)-p)} = \alpha_c, \tag{5}$$

such that if $v > T(p,h,\alpha_c)$, then the consumer will purchase 2 units at the beginning of Period 1, and if $p < v \leq T(p,h,\alpha_c)$, then he will purchase 1 units. ■

Theorem 1 identifies a threshold policy for the consumers, which is quite intuitive. When the consumer valuation is low, then he will not buy any products. When the consumer valuation is median, then he only buy one unit product for each period if there is any. When the consumer valuation is high, then he will stockpile one unit of product in Period 1 to prevent its unavailability in Period 2. Next we examine some properties of the threshold $T(p,h,\alpha_c)$. The proofs are in the Appendix.

PROPOSITION 1. *$T(p,h,\alpha_c)$ is increasing in p,h and α_c.*

Proposition 1 indicates that the consumers are more likely to stockpile products when the selling price or the consumer's holding cost is lower, or the consumer is less confident on that he can get what he purchases in Period 2.

Next we investigate how $T(p,h,\alpha_c)$ changes with the consumers' degree of risk aversion. According to Pratt (1964), an increase in the degree of risk aversion can be represented by an increasing concave transformation. Let $W(\cdot)=\varphi\big(U(\cdot)\big)$, where φ is an increasing concave function with $\varphi(0)=0$. Thus utility function $W(\cdot)$ represents a higher degree of risk aversion. Let T_U, T_W be the thresholds corresponding to $U(\cdot)$, $W(\cdot)$ respectively. Then we have the following proposition.

PROPOSITION 2. *$T_U \geq T_W$, that is, a consumer with higher degree of risk aversion is more likely to purchase 2 units at the beginning of Period 1.*

Proposition 2 shows that the consumers are more likely to stockpile products if they are more risk averse.

5. Equilibrium analysis

In this section, we conduct equilibrium analysis to study the retailer's inventory policy and the cost of ignoring consumer behaviors. To study the strategic interaction between the retailer and consumers, and those among the consumers, we will adopt the notion of the rational expectation (RE) equilibrium. This equilibrium concept is commonly used in the strategic consumer behavior literature (See Liu and Ryzin 2008, Su and Zhang 2009 for examples). For given consumers' belief over fulfill rate α_c and the retailer's order quantities at the beginning of Periods 1 and 2 (i.e., Q_1, Q_2), the retailer's expected profit is

$$\begin{aligned}\Pi(Q_1,Q_2,\alpha_c) = {}& p\min\Big\{N\overline{F}(p)+N\overline{F}\big(T(p,h,\alpha_c)\big),Q_1\Big\} - H\big[Q_1-N\overline{F}(p)-N\overline{F}\big(T(p,h,\alpha_c)\big)\big]^+\\ &+\beta p\min\Big\{Q_2+\big[Q_1-N\overline{F}(p)-N\overline{F}(T(p,h,\alpha_c))\big]^+,\ NF\big(T(p,h,\alpha_c)\big)-NF(p)\Big\}\\ &+(1-\beta)p\big[Q_1-N\overline{F}(p)-N\overline{F}(T(p,h,\alpha_c))\big]^+, \end{aligned} \tag{6}$$

where β is the probability of supply success and $T(p,h,\alpha_c)$ is defined as in (5). In Eqn. (6), the first term represents the revenue in Period 1, the second term stands for the holding cost, the third term represents the expected revenue when supply is successful and the last term represents the expected revenue under supply disruption. For any given α_c, let $Q_1^*(\alpha_c)$, $Q_2^*(\alpha_c)$ be the optimal order quantities maximizing (6). Clearly, it is optimal for the retailer to order $Q_2^*(\alpha_c)=2N\overline{F}(p)-\max\Big\{N\overline{F}(p)+N\overline{F}\big(T(p,h,\alpha_c)\big),Q_1^*(\alpha_c)\Big\}$ at the beginning of Period 2. Thus, in the rest of this paper, we ignore the decision Q_2 and only consider the decision Q_1. Next we introduce the definition of Rational Expectation (RE) equilibrium.

DEFINITION 1. A RE equilibrium satisfies the following conditions:

(i) For given belief α_c, the optimal purchasing policy of any consumer who possesses valuation v is $z_1 = z_1^*(\alpha_c)$, $z_2 = z_2^*(\alpha_c)$, where $z_1^*(\alpha_c)$ and $z_2^*(\alpha_c)$ are defined in Theorem 1.

(ii) For given belief α_c, the retailer's optimal order quantities satisfy $Q_1 = Q_1^*(\alpha_c)$, $Q_2 = Q_2^*(\alpha_c) = 2N\overline{F}(p) - \max\left\{N\overline{F}(p) + N\overline{F}\big(T(p,h,\alpha_c)\big), Q_1^*(\alpha_c)\right\}$.

(iii) The beliefs of all the consumers α_c are consistent with the realized fulfill rate α, i.e.,

$$\alpha_c = \alpha \triangleq \beta + (1-\beta)\min\left\{\frac{\Big[Q_1^*(\alpha_c) - N\big(\overline{F}(p) + \overline{F}(T(p,h,\alpha_c))\big)\Big]^+}{N\Big[F\big(T(p,h,\alpha_c)\big) - F(p)\Big]},\ \ 1\right\}. \tag{7}$$

In Eqn. (7), the term $\Big[Q_1^*(\alpha_c) - N\big(\overline{F}(p) + \overline{F}(T(p,h,\alpha_c))\big)\Big]^+$ stands for the retailer's inventory at the beginning of Period 2 and the term $N\Big[F\big(T(p,h,\alpha_c)\big) - F(p)\Big]$ is the number of consumers who want to buy the product in Period 2. Since it requires $\alpha_c = \alpha$ in the equilibrium, we use α to replace α_c hereafter. Furthermore, for simplicity, we write $Q_1^*(\alpha_c)$ as Q_1^*

5.1 Retailer's inventory policy

In this subsection, we discuss the retailer's inventory policy under RE equilibrium. First, we can show that the retailer's first order quantity Q_1^* is no greater than $2N\overline{F}(p)$, which is the total demand of both periods. Second, we can show that the retailer's first order quantity Q_1^* is no less than $N(\overline{F}(p) + \overline{F}(T(p,h,\beta)))$, the demand of Period 1 if retailer carries no inventory at the end of Period 1. Otherwise, the retailer can increase its net profit by increasing Q_1^* to $N(\overline{F}(p) + \overline{F}(T(p,h,\beta)))$, which increases the sales and corresponding revenue in Period 1, while does not change other revenue or cost. As a result, to maximize the retailer's expected profit, it must hold that $Q_1^* \in \Big[N\big(\overline{F}(p) + \overline{F}(T(p,h,\beta))\big), 2N\overline{F}(p)\Big]$. Therefore, the RE equilibrium can be obtained by solving the following optimization problem:

$$\max_{Q_1,T,\alpha}\ \Pi = 2pN\overline{F}(p)\beta + pQ_1(1-\beta) - H[Q_1 - N(\overline{F}(p) + \overline{F}(T))] \tag{8}$$

$$s.t.\ \ \alpha = \beta + (1-\beta)\frac{Q_1 - N[\overline{F}(p) + \overline{F}(T)]}{N[F(T) - F(p)]}, \tag{9}$$

$$\alpha U(T-p) = U(T-p-h), \tag{10}$$

$$Q_1 \in [N(\overline{F}(p) + \overline{F}(T(p,h,\beta))), 2N\overline{F}(p)],\ \ \alpha \in [\beta, 1]. \tag{11}$$

Objective function (8) is equivalent to the retailer's expected profit in Eqn. (6) with $Q_1 \in [N(\overline{F}(p) + \overline{F}(T(p,h,\beta))), 2N\overline{F}(p)]$ and $Q_2 = 2N\overline{F}(p) - Q_1$. Constraint (9) is by the rational expectation constraint, Constraint (10) comes from the definition of the threshold T and Constraint (11) is due to the discussion before Eqn. (8). Here we omit the constraint $T \in [T(p,h,\beta), +\infty)$, which is implied by

Constraints (9)-(11). Submitting (9) and (10) into the objective function (8), we have the following problem:

$$\max_{T} \quad \Pi = 2pN\overline{F}(p) - N\left[p - \frac{H\beta}{1-\beta} - \left(p - \frac{H}{1-\beta}\right)\frac{U(T-p-h)}{U(T-p)}\right] \cdot [F(T) - F(p)] \tag{12}$$

$$s.t. \;\; T \in \left[T(p,h,\beta), +\infty\right). \tag{13}$$

Once the optimal solution to Problem (12)-(13), T^*, is found, we can easily calculate the realized fulfill rate and the optimal order quantity Q_1^*:

$$\alpha^* = U(T^* - p - h)/U(T^* - p), \tag{14}$$

$$Q_1^* = \frac{\alpha^* - \beta}{1-\beta} N[F(T^*) - F(p)] + N[\overline{F}(T^*) + \overline{F}(p)]. \tag{15}$$

To facilitate analysis, we assume in the rest of the paper that the consumers possess a power utility function $U(x) = x^\gamma$ $(0 < \gamma \le 1)$, which is a common utility function representing risk aversion in economics and operations management literature (e.g., Liu and Ryzin 2008). Lower value of γ represents higher degree of risk aversion. In addition, we assume consumer valuation are uniformly distributed on $[0, \overline{v}]$, where average consumer valuation is $\overline{v}/2$. Here $\overline{v}$ can be interpreted as a measure of consumers' desirability for the product. To avoid trivial cases, we assume that $\overline{v} > p$. By simple calculation, we have $T(p,h,\alpha) = p + h/(1 - \alpha^{\frac{1}{\gamma}})$ and Problem (12)-(13) comes into

$$\max_{T} \quad \Pi = \frac{2Np(\overline{v}-p)}{\overline{v}} - N \cdot \left\{ \min\left(\frac{T}{\overline{v}}, 1\right) - \frac{p}{\overline{v}} \right\} \cdot \left\{ H + \left(p - \frac{H}{1-\beta}\right)\left[1 - \left(1 - \frac{h}{T-p}\right)^\gamma\right] \right\} \tag{16}$$

$$s.t. \;\; T \in \left[p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}, +\infty\right). \tag{17}$$

Some interesting questions are: (1) In what conditions should the retailer carry inventory at the end of Period 1 to increase the fulfill rate in Period 2? (2) How much inventory should the retailer carry? (3) How does the consumers' risk preference affects the retailer's inventory decision? The following two theorems answer these questions.

Theorem 2. *Suppose the consumers are risk neutral, i.e., $\gamma = 1$. Then the retailer's optimal order quantity Q_1^* and the realized fulfill rate are*
(i) $Q_1^* = N[\overline{F}(p) + \overline{F}(p + \frac{h}{1-\beta})]$, $\alpha^* = \beta$, if $(\beta, \overline{v}) \in \Omega_1^n$;
(ii) $Q_1^* = 2N\overline{F}(p)$, $\alpha^* = 1$, if $(\beta, \overline{v}) \in \Omega_2^n$,
where $\Omega = \{(\beta, \overline{v}) | 0 \le \beta \le 1, p \le \overline{v} \le +\infty\}$, $\Omega_2^n = \{(\beta, \overline{v}) | \beta < 1 - \frac{H}{p}, \overline{v} < \frac{p(H+h)}{H}\}$ and $\Omega_1^n = \Omega - \Omega_2^n$.

Proof. Case (a): $\beta > 1 - H/p$. In this case, we have $p < H/(1-\beta)$. It can be easily verified that $\Pi(T)$ in (16) is decreasing with respect to T, so it is optimal to choose $T^* = T(p,h,\beta)$. Thus, in this case, $\alpha^* = \beta$, $Q_1^* = N[\overline{F}(T(p,h,\beta)) + \overline{F}(p)] = N[2 - \frac{2p}{\overline{v}} - \frac{h}{(1-\beta)\overline{v}}]$.

Case (b): $\beta \leq 1 - H/p$. Problem (16)-(17) can be split into two subproblems:

$$\max_{T} \quad \Pi_1(T) = \frac{2Np(\overline{v}-p)}{\overline{v}} - \frac{N(\overline{v}-p)}{\overline{v}} \cdot \left\{H + \left(p - \frac{H}{1-\beta}\right)\left[1 - (1 - \frac{h}{T-p})^{\gamma}\right]\right\} \tag{18}$$

$$s.t. \quad T \in [\overline{v}, +\infty), \tag{19}$$

$$\max_{T} \quad \Pi_2(T) = \frac{2Np(\overline{v}-p)}{\overline{v}} - \frac{N(T-p)}{\overline{v}} \cdot \left\{H + \left(p - \frac{H}{1-\beta}\right)\left[1 - \left(1 - \frac{h}{T-p}\right)^{\gamma}\right]\right\} \tag{20}$$

$$s.t. \quad T \in \left[p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}, \overline{v}\right]. \tag{21}$$

First, let's consider the optimal solution of Subproblem (18)-(19). It is clearly that $\Pi_1(T)$ defined in (19) is increasing with respect to T. Thus, the optimal solution to Problem (18)-(19) is $T_1^* = +\infty$ and the optimal value is

$$\Pi_1^* = \Pi_1(T_1^*) = \frac{2Np(\overline{v}-p)}{\overline{v}} - \frac{NH(\overline{v}-p)}{\overline{v}}. \tag{22}$$

Now we consider Subproblem (20)-(21). Taking first and second derivatives of $\Pi_2^{'}(T)$ in (20), we have

$$\Pi_2^{'}(T) = -\frac{N}{\overline{v}}\left\{H - \left(p - \frac{H}{1-\beta}\right)\left[\left(1 - \frac{h}{T-p}\right)^{\gamma-1}\left(1 - \frac{(1-\gamma)h}{T-p}\right) - 1\right]\right\}. \tag{23}$$

and $\Pi_2^{''}(T) < 0$. Since $\gamma = 1$, we have $\Pi_2^{'}(T) < 0$ for all $T > p + h$, which indicates that the optimal solution for Subproblem (20)-(21) is $T_2^* = p + \frac{h}{1-\beta}$, and the optimal value is

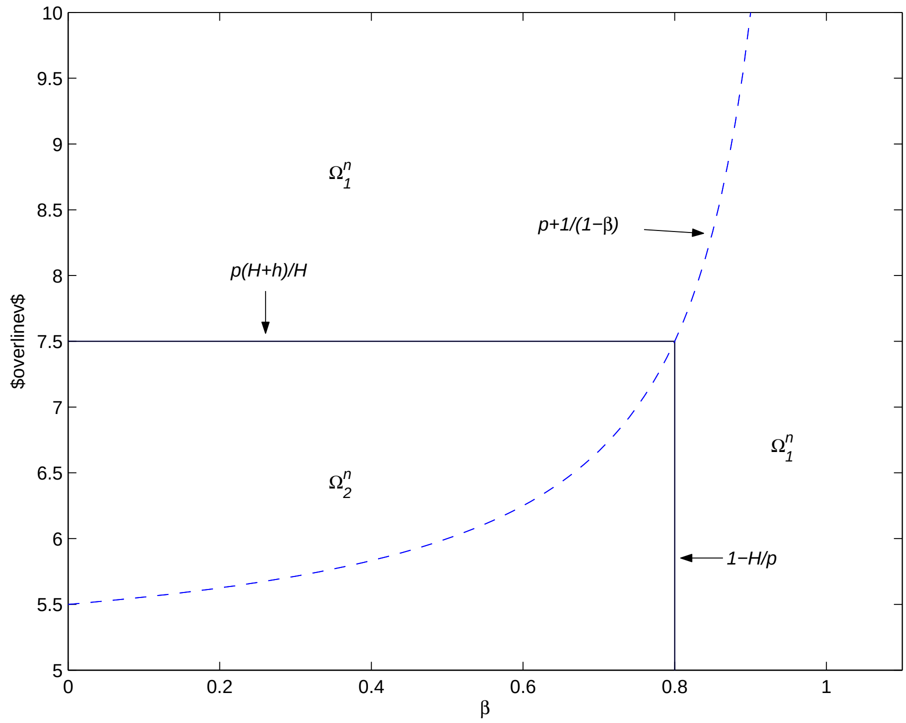
$$\Pi_2^* = \Pi_2(T_2^*) = \frac{2Np(\overline{v}-p)}{\overline{v}} - \frac{Nph}{\overline{v}}. \tag{24}$$

If $p \leq \overline{v} < p + \frac{h}{1-\beta}$, then Subproblem (20)-(21) vanishes and the optimal solution is $T^* = +\infty$. If $p + \frac{h}{1-\beta} \leq \overline{v} < \frac{p(H+h)}{H}$, then $\Pi_1^* > \Pi_2^*$, which implies $T^* = T_1^* = +\infty$. If $\overline{v} \geq \frac{p(H+h)}{H}$, then $\Pi_1^* \leq \Pi_2^*$ and $T^* = T_2^* = p + \frac{h}{1-\beta}$. ■

Theorem 2 states that, when facing risk neutral consumers, the retailer will carry inventory when both the supply reliability and consumers' desirability for the product are low. Otherwise, it won't carry inventory. We will provide a explanation later (following Theorem 3). Fig. 1 demonstrates the retailer's inventory policy when facing risk neutral consumers under the case of $p = 5$, $H = 1$ and $h = 0.5$.

PROPOSITION 3. *For any consumer valuation distribution $F(v)$ and any increasing concave utility function $U(x)$, if $(1-\beta)p < H$ then the optimal order quantity and fulfill rate are $Q_1^* = N[\overline{F}(T(p,h,\beta)) + \overline{F}(p)]$ and $\alpha^* = \beta$.*

Figure 1 The optimal inventory policy for the retailer facing risk neutral consumers



Proposition 3 indicates that the conclusion that the retailer shouldn't carry inventory when supply reliability is low is very robust, even with only mild conditions the distribution of consumer valuation and the consumers' utility function.

Before providing the Theorem 3, we introduce some notation. Let β_t be the solution of

$$H-(p-\frac{H}{1-\beta})[\gamma\beta^{1-\frac{1}{\gamma}}+(1-\gamma)\beta-1]=0,$$

T^o be the solution of $\Pi_2^{'}(T)=0$ in (23) and

$$v_t(\beta)=\begin{cases} p+\frac{T^o-p}{H}\{H+(p-\frac{H}{1-\beta})[1-(1-\frac{h}{T^o-p})^{\gamma}]\}, & if\ \beta<\beta_t, \\ p+\frac{ph(1-\beta)}{H(1-\beta^{\frac{1}{\gamma}})}, & if\ \beta\geq\beta_t. \end{cases}$$

Denote

$$\Omega=\Big\{(\beta,\overline{v})|0\leq\beta\leq 1, p\leq\overline{v}\leq+\infty\Big\},$$
$$\Omega_2^v=\Big\{(\beta,\overline{v})|0\leq\beta<1-\frac{H}{p},\overline{v}<v_t(\beta)\Big\},$$
$$\Omega_3^v=\Big\{(\beta,\overline{v})|0\leq\beta\leq\beta_t,\overline{v}>v_t(\beta)\Big\},$$
$$\Omega_1^v=\Omega-\big(\Omega_2^v\cup\Omega_3^v\big).$$

THEOREM 3. *Suppose the consumers are risk averse with $0<\gamma<1$. Then the retailer's optimal order quantity and fulfill rate are as follows:*

(i) If $(\beta,\overline{v}) \in \Omega_1^v$*, then* $Q_1^* = N\Big[\overline{F}(p) + \overline{F}\big(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\big)\Big]$, $\alpha^* = \beta$*;*

(ii) If $(\beta,\overline{v}) \in \Omega_2^v$*, then* $Q_1^* = 2N\overline{F}(p)$, $\alpha^* = 1$*;*

(iii) If $(\beta,\overline{v}) \in \Omega_3^v$*, then*

$$\begin{aligned} Q_1^* &= \frac{\alpha^* - \beta}{1-\beta} N\big[F(T^o) - F(p)\big] + N\big[\overline{F}(T^o) + \overline{F}(p)\big] \\ &\in \Big(N\Big[\overline{F}(p) + \overline{F}(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}})\Big], 2N\overline{F}(p)\Big), \\ \alpha^* &= \frac{(T^o - p)^\gamma}{(T^o - p - h)^\gamma} \in (\beta, 1). \end{aligned}$$

Proof. We divide our proof in three cases: (a) $\beta > 1 - \frac{H}{p}$, (b) $\beta_t < \beta \le 1 - \frac{H}{p}$, (c) $0 < \beta \le \beta_t$.

For Case (a), we have $Q_1^* = N[\overline{F}(T(p,h,\beta)) + \overline{F}(p)]$ and $\alpha^* = \beta$ by Proposition 3.

For Case (b), in which $\beta > \beta_t$, since

$$X(\beta,\gamma,p,H) = \Pi'(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}) = -\frac{N}{\overline{v}}\{H - (p - \frac{H}{1-\beta})[\gamma\beta^{1-\frac{1}{\gamma}} + (1-\gamma)\beta - 1]\} < 0, \tag{25}$$

we have $T^o < p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}$. (It can be verified that $X(\beta,\gamma,p,H)$ is decreasing with respect to β, $lim_{\beta\to 0} X(\beta,\gamma,p,H) = +\infty$ and $X(1 - H/p, \gamma, p, H) = -NH/\overline{v} < 0$. Thus, β_t does lie in $(0, 1 - \frac{H}{p})$.) Recall that Problem (16)-(17) can be split into Subproblems (18)-(19) and (20)-(21). With a similar arguement as in the Proof of Theorem 2, we have the optimal solution to Subproblem (18)-(19) is $T_1^* = +\infty$ and the optimal value is Π_1^* in (22). And the optimal solution to Subproblem (20)-(21) is $T_2^* = p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}$ and the optimal value is $\Pi_2^* = \Pi_2(T_2^*) = \frac{2Np(\overline{v}-p)}{\overline{v}} - \frac{Nph(1-\beta)}{\overline{v}(1-\beta^{\frac{1}{\gamma}})}$.

If $\overline{v} < p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}$, then Subproblem (20)-(21) vanishes and the optimal solution is $T^* = T_1^* = +\infty$. If $p + \frac{h}{1-\beta^{\frac{1}{\gamma}}} \le \overline{v} < v_t(\beta)$, then $\Pi_1^* > \Pi_2^*$, which indicates $T^* = T_1^* = +\infty$. If $\overline{v} \ge v_t(\beta)$, then $\Pi_1^* \le \Pi_2^*$, which implies that $T^* = T_2^* = p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}$.

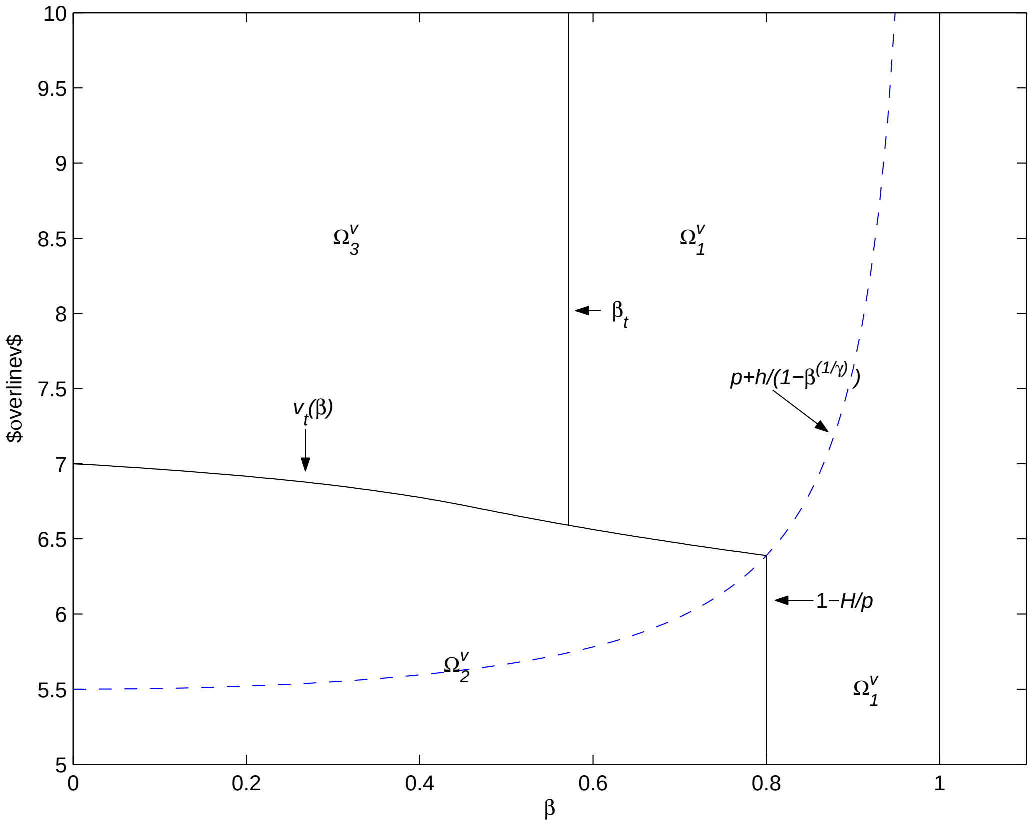
In Case (c), it can be verified that $T^o \ge p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}$. The optimal solution of Subproblem (20)-(21) changes to $T_2^* = min\{T^o, \overline{v}\}$. If $\overline{v} < T^o$, then $T_2^* = \overline{v}$. Thus, $\Pi_1^* = \Pi_1(+\infty) > \Pi_1(\overline{v}) = \Pi_2(\overline{v}) = \Pi_2^*$, which indicates that $T^* = T_1^* = +\infty$. If $T^o \le \overline{v} < v_t(\beta)$, then $T_2^* = T^o$ (implied by $T^o \le \overline{v}$) and $\Pi_1^* = \Pi_1(+\infty) > \Pi_2(T^o) = \Pi_2^*$ (implied by $\overline{v} < v_t(\beta)$). Thus, $T^* = T_1^* = +\infty$. If $\overline{v} \ge v_t(\beta)$, then $T_2^* = T^o$ and $\Pi_1^* = \Pi_1(+\infty) \le \Pi_2(T^o) = \Pi_2^*$. Thus, $T^* = T_2^* = T^o$. Summarizing the above conclusions, one can come to this theorem. ■

Theorem 3 shows that, facing risk averse consumers, the retailer shouldn't carry inventory at the end of Period 1 when the supply reliability is high, and should carry inventory when the supply reliability is low. In particular,when supply reliability is low, the retailer should carry inventory at the end of Period 1 to cover all demands of Period 2 if the consumers' desirability for the product is low, and should carry less inventory to cover partial demands of Period 2. The interpretation is as follows. When making order decision, the retailer considers two types of cost: the inventory holding cost and the stockout cost due to supply disruption in Period 2. If the retailer increases the inventory at the end of Period 1, he will lower the stockout cost $(1-\beta)p$, but incur a higher holding cost. Furthermore, increasing the inventory at the end of Period 1 will increase the fulfill rate α in Period 2, which brings down the number of consumers who purchase two units of products in Period 1. Hence, increasing inventory may increase the retailer's holding cost by more than H (The extra holding cost due to increasing inventory depends on interactions of consumers' risk preference and desirability for the product). If the supply reliability is high, the reduced stockout cost by carrying one unit of inventory $(1-\beta)p$ is lower than the increased holding cost. Thus, the retailer has no incentive to carry inventory at the end of Period 1. Now we consider the case that the supply reliability is low, whereby the stockout cost is relatively high. When consumers' desirability for the product is low, there are not many consumers who purchase two units of products in Period 1. Even if the retailer carries inventory to cover all demands of Period 2 (which increases the fulfill rate in Period 2 to 1), the demand of Period 1 doesn't decrease significantly. Thus, the extra holding cost due to reducing the consumers' stockpiling incentives and shrinking the demands in Period 1 is relatively low. Hence, the retailer carries inventory at the end of Period 1 to cover all the demands in Period 2. When consumers' desirability for the product is high, there are a lot of consumers stockpiling in Period 1. Thus, the demand shrinks more dramatically and the holding cost increases more significantly when the inventory at the end of Period 1 becomes more. There is a balance point of inventory at the end of Period 1, where the reduced stockout cost equals to the marginal holding cost incurred. Therefore, the retailer tends to carry less inventory to only cover partial demands of Period 2 when the consumers' desirability for product is high.

Comparing Theorems 2 and 3, we find that consumers' risk preferences do influence the retailer's ordering decision. Interestingly, we find that when supply reliability is low and consumers' desirability for the product is high, the retailer does not carry any inventory if consumers are risk neutral, but it carries inventory if consumers are risk averse. This is because, the consumers are less willing to stockpile the product when their degrees of risk aversion decrease. When the retailer increases the inventory at the end of Period 1 (which leads the fulfill rate in Period 2 higher), in comparison with risk averse case, more risk neutral consumers switch to purchasing only one unit of product from purchasing two in Period 1. This means that increasing inventory shrinks demand of Period 1 more under the risk neutral case than the risk averse one, i.e., carrying inventory incurs more holding cost under the risk neutral case than the risk averse one, while the reduced stockout costs under the two cases are the same. Therefore, the retailer tends to carry more inventory when the consumers are more risk averse.

Next we provide a numerical example to demonstrate the results of Theorem 3. In this example, we let $p=5$, $H=1$, $h=0.5$, $\gamma=0.5$ and the result is demonstrated by Fig. 2. From Fig. 2, we can

Figure 2 **The optimal inventory policy for the retailer facing risk averse consumers**



observe that the two-dimensional space of $(\beta, \overline{v})$ is divided into 3 regions. In the region Ω_1^v, the retailer doesn't carry any inventory at the end of Period 1 and the realized fulfill rate is β. In the region Ω_2^v, the retailer carries inventory at the end of Period 1 to cover all the demands in Period 2 and the realized fulfill rate is 1. In the region Ω_3^v, the retailer carries some inventory, but doesn't cover all demands in Period 2, which indicates that the realized fulfill rate is between β and 1.

PROPOSITION 4. *(i) β_t is decreasing in H, γ and increasing in p;*
(ii) T^o is decreasing in H,β and increasing in p.

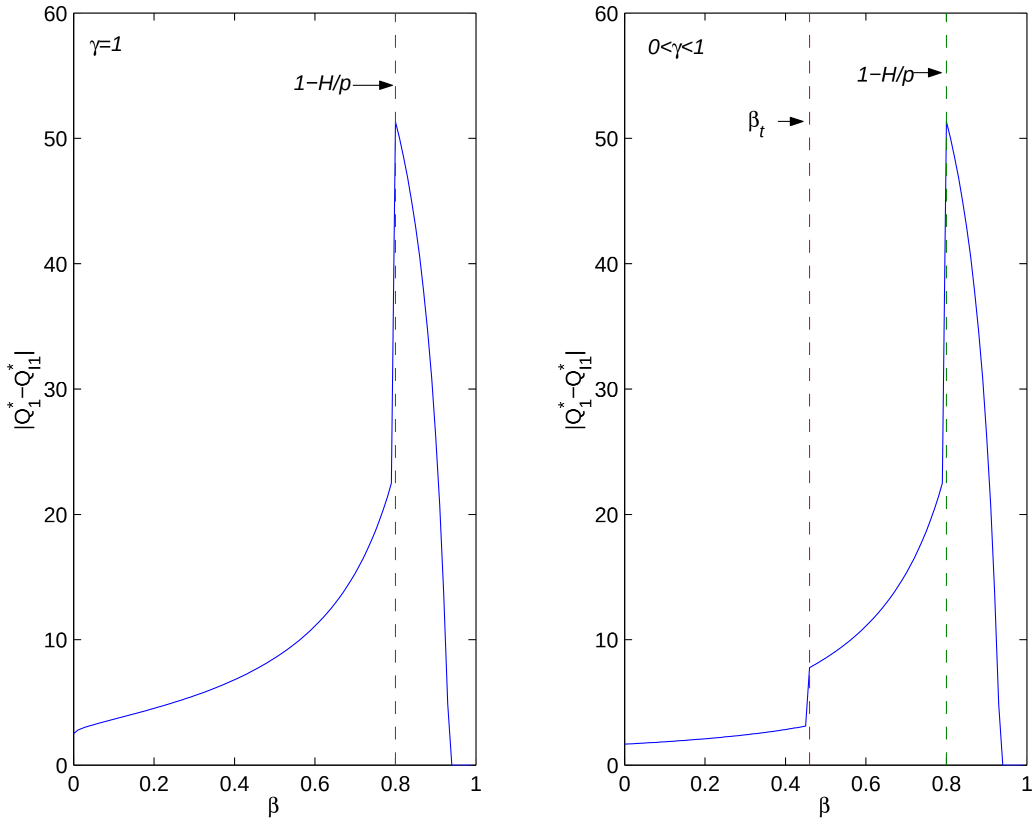
Proposition 4 characterize monotone properties of β_t and T^o with respect to some parameters.

5.2 Cost of ignoring consumer behaviors

In this subsection, we try to answer the following question: In what situations will the retailer's ignorance of consumer behaviors have severe consequences? To this end, we first characterize the optimal order quantity when the retail ignores consumer panic buying behaviors. With this assumption, the retailer choose Q_{I1} and Q_{I2} (the subscript I represents the case of ignoring consumer behavior) to maximize the expected profit

$$\Pi_I(Q_{I1}, Q_{I2}) = pQ_{I1} - H[Q_{I1} - N\overline{F}(p)] + \beta p Q_{I2}. \tag{26}$$

Figure 3 The inventory deviations of ignoring consumer behaviors



Clearly, for any given Q_{I1}, it is optimal to order $Q_{I2} = 2N\overline{F}(p) - Q_{I1}$. Thus, we only consider the decision Q_{I1}. By simple calculation, we show that if we replace Q_{I2} by $2N\overline{F}(p) - Q_{I1}$, the objective function in Equ. (26) is linear is Q_{I1}. Thus the optimal first order quantity is $Q^*_{I1} = N\overline{F}(p)$, if $\beta > 1 - \frac{H}{p}$; and $Q^*_{I1} = 2N\overline{F}(p)$, if $\beta \leq 1 - \frac{H}{p}$. The next following theorem compares the optimal order quantities of considering consumer behaviors and ignoring consumer behaviors.

THEOREM 4. *(a) Suppose* $\gamma = 1$.

(i) $Q^*_1 = Q^*_{I1}$, *if* $(\beta, \overline{v}) \in \Omega^n_2$ *and* $(\beta, \overline{v}) \in \{(\beta, \overline{v}) \in \Omega^n_1 | \overline{v} < p + \frac{h}{1-\beta}\}$;

(ii) $Q^*_1 \neq Q^*_{I1}$, *if* $(\beta, \overline{v}) \in \{(\beta, \overline{v}) \in \Omega^n_1 | \overline{v} \geq p + \frac{h}{1-\beta}\}$. *Furthermore,* $|Q^*_1 - Q^*_{I1}|$ *is increasing in* β *when* $\beta \in (0, 1 - \frac{H}{p})$, *and decreasing in* β *when* $\beta \in (1 - \frac{H}{p}, 1 - \frac{h}{\overline{v}-p})$.

(b) Suppose $0 < \gamma < 1$.

(i) $Q^*_1 = Q^*_{I1}$, *if* $(\beta, \overline{v}) \in \Omega^v_2$ *and* $(\beta, \overline{v}) \in \{(\beta, \overline{v}) \in \Omega^v_1 | \overline{v} < p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\}$;

(ii) $Q^*_1 \neq Q^*_{I1}$, *if* $(\beta, \overline{v}) \in \Omega^v_3$ *and* $(\beta, \overline{v}) \in \{(\beta, \overline{v}) \in \Omega^v_1 | \overline{v} \geq p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\}$. *Furthermore,* $|Q^*_1 - Q^*_{I1}|$ *is increasing in* β *when* $\beta \in (\beta_t, 1 - \frac{H}{p})$, *and decreasing in* β *when* $\beta \in (1 - \frac{H}{p}, (1 - \frac{h}{\overline{v}-p})^{\gamma})$.

Theorem 4 indicates that the retailer can ignore consumer behaviors when either supply reliability is high or the consumers' desirability for the product is low. Otherwise, the retailer should take consumer behavior into consideration. Recall Figs. 1 (or 2) when $(\beta, \overline{v})$ lies in the area below the curve $\max\{p(H+h)/H, p+h/(1-\beta)\}$ (or, $\max\{v_t(\beta), p+h/(1-\beta^{\frac{1}{\gamma}})\}$), the retailer can ignore consumer behaviors. When $(\beta, \overline{v})$ lies in the area above it, the retailer should take consumer behaviors into consideration. Theorem 4 also suggests that the decision deviation of ignoring consumer behaviors is the most significant when supply reliability is round $1-\frac{H}{p}$. Fig. 3 illustrates these results in the case of $p=5$, $H=1$, $h=0.5$ $N=100$ and $\overline{v}=10$.

Next we discuss the profit loss of ignoring consumer behavior and how it changes with model parameters. Define the percentage of profit loss (PL) caused by ignoring consumer behavior as

$$PL = \frac{\Pi^* - \Pi_I^*}{\Pi^*}, \tag{27}$$

where Π^* is the retailer's optimal profit when considering consumer behavior and Π_I^* is the retailer's profit when it ignores consumer behavior but consumer behavior actually exists, i.e., the corresponding profit when choosing Q_{I1}^*. When the retailer's order quantity is less than the demand in Period 1, obviously there will be lost sales. In this case, consumer arrival sequence does affect the retailer's expected profit. Here we assume a uniform arrival sequence for the consumers (please refer to the next section for details of the assumption). When stockout occurs in Period 1, the retailer's expected profit is calculated based on this assumption. Next we provide a numerical example to demonstrate how PL changes with supply reliability, consumers' desirability for the product and consumers' degree of risk aversion. The example is based on the following parameter values: $p=5$, $H=1$, $h=0.5$.

Table 2. The percentage of profit loss caused by ignoring consumer behavior v.s. supply reliability and consumers' desirability for the product; $\gamma=0.5$.

$\overline{v}$ \ PL \ β	0.15	0.35	0.55	0.75	0.85	0.9	0.95	1
6	0	0	0	0	0	0	0	0
8	0.038	0.042	0.05	0.055	0.158	0.058	0	0
10	0.064	0.066	0.071	0.074	0.22	0.173	0	0
12	0.074	0.076	0.079	0.081	0.241	0.208	0.109	0
14	0.08	0.081	0.084	0.085	0.253	0.225	0.156	0

Table 3. The percentage of profit loss caused by ignoring consumer behavior v.s. consumers' degree of risk aversion; $\overline{v} = 10$.

β \ PL \ γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.75	0.088	0.085	0.081	0.078	0.074	0.069	0.065	0.061	0.057
0.85	0.267	0.256	0.245	0.233	0.22	0.206	0.191	0.174	0.157

We have the following observations:

First, when supply reliability is either high or low, the percentage of profit loss caused by ignoring consumer behavior is relatively low. Nevertheless, when supply reliability is median (round $1 - H/p$), the percentage of profit loss can be very high (it can be as large as 25% as shown in Table 2). This can be interpreted as follows. When supply reliability is high, the consumers' stockpiling behavior is not significant. Thus, ignoring consumer behavior will not cause many losses. When supply reliability is low, many consumers tend to purchase two units of product in Period 1. Hence, if the retailer considers consumer behavior, the retailer will order many products before Period 1 to satisfy the demand, or even order more into inventory to hedge the risk of supply disruption in Period 2. Coincidentally, it is optimal for the retailer to make an order before Period 1 to satisfy the demands of the two periods, if the retailer ignores consumer behavior. Therefore, when supply reliability is low, the cost of ignoring consumer behavior is also not high. When the supply reliability is intermediate, ignoring consumer behavior costs the retailer much, because ignoring consumer behavior makes the retailer incur higher holding cost (if $\beta < 1 - H/p$), or higher stockout cost ($\beta > 1 - H/p$). As we can see, stockout cost p is much higher than holding cost H. Thus, the percentage of profit loss is much higher when β approaches $1 - H/p$ from above than from below.

Second, the percentage of profit loss caused by ignoring consumer behavior is increasing with respect to the consumers' desirability for the product. This is very intuitive. When the consumers' desirability for the product becomes higher, then the consumer behavior becomes more significant. Thus, the percentage of profit loss caused by ignoring consumer behavior is larger.

Finally, the percentage of profit loss caused by ignoring consumer behavior is higher when consumers' degree of risk aversion is higher, the intuition of which is similar as above.

6. Capacity constraint and Fixed Quota Policy

In this section, we consider that the retailer has a capacity constraint: its order cannot exceed K units during each period. There are various reasons for us to consider the capacity constraint, e.g., the vendor has limited product supply, or the retailer has limited storage space. We examine the impact of limited capacity on the retailer and consumers' decisions. Furthermore, we study fixed quota policy (e.g., limiting purchase to one unit per customer) and its effectiveness in dealing with consumer panic buying.

6.1 Capacity Constraint

We assume that the capacity K is no less than one-period demand without consumer stockpiling, i.e., $K \geq N\overline{F}(p)$, otherwise the case becomes trivial.

First, consider the case of $K \geq N\Big(\overline{F}(p) + \overline{F}\big(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\big)\Big)$. The retailer can satisfies all consumer demands in Period 1. The retailer's problem is whether to carry inventory to increase the fulfill rate in Period 2 and how much inventory to carry. In this case, we find that the managerial insights are similar as in Section 5, so we do not repeat here.

Next, consider the case of $N\overline{F}(p) \leq K < N\Big(\overline{F}(p) + \overline{F}\big(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\big)\Big)$. The retailer cannot meet all consumer demands in Period 1, as a result some of the consumers can not get what they purchase in Period 1. In this situation, we assume that the consumer will go to another store to purchase an alternative one. However, in Period 2, the consumers who don't get products in Period 1 is assumed to be loyal to the retailer and will still come to the retailer to buy products. One may argue that the consumers purchasing decisions will change since they cannot get products in Period 1 for sure. However, here we assume that their purchasing decisions do not change. There are two reasons: (1) consumers usually don't know how much capacity the retailer has, so they may not perceive the possibilities of stockout in Period 1; (2) Given other consumers' purchasing decision, one consumer will not get better by changing his decision when there is capacity constraint than there is no such constraint for the retailer. Because given other consumers' purchasing decisions, buying more or less products will not affect the probability of one consumer getting his demand. To simplify our exposition, denote $K = \delta N\overline{F}(p)$. Thus, we focus on the case of

$$1 \leq \delta < 1 + \frac{\overline{F}\big(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}}\big)}{\overline{F}(p)}. \tag{28}$$

From the retailer point of view, there are two types of consumers: consumers purchasing 1 product in Period 1 (we refer to this type of consumers as Type-1 consumer), and consumers purchasing 2 product in Period 1 (Type-2 consumer). Under Constraint (28), the retailer cannot serve all the consumers in Period 1. Although the retailer always sells K units of product in Period 1, the arrival sequence of consumers do affect the demand of Period 2. If all the Type-2 consumers arrive before Type-1 consumers, then the demand in Period 2 will be lower in comparison with the case that Type-1 consumers arrive before Type-2 consumers. In this paper, we assume an uniform arrival for consumers. (Similar analysis in the section can be conducted for other arrival patterns, e.g., consumers with higher valuations arrive earlier. The major managerial insights are same.) The uniform arrival form can be explained as follows. Suppose the proportion of Type-2 consumers among the whole consumer population is ξ and so far x consumers have arrived. Then among the x consumers, ξx are Type-2 consumers and $(1-\xi)x$ are Type-1. Under this uniform arrival pattern, for a large population of consumers (i.e., large N), the number of Type-2 consumers served in Period 1 is

$$\frac{K}{N\overline{F}(p)+N\overline{F}\Big(p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}\Big)}\cdot N\overline{F}(p)\cdot\frac{\overline{F}\Big(p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}\Big)}{\overline{F}(p)}=K\frac{\overline{F}\Big(p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}\Big)}{\overline{F}(p)+\overline{F}\Big(p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}\Big)}. \tag{29}$$

The first term in left-hand-side (LHS) of Eqn. (29) is the total number of available products over total demand of Period 1, thus it means the served percentage of total demand. The second term in LHS of Eqn. (29) stands for the total number of consumers, including Type-1 and Type-2 consumers. The product of the first two terms stands for the total number of served consumers with capacity K. Here we use the served percentage of total demand to approximate the served percentage of all the consumers based on the assumptions of the uniform arrival pattern and a large population of consumers. The third term in LHS of Eqn. (29) is the percentage of the Type-2 consumer among all consumers. Denote

$$\theta=\frac{\overline{F}\Big(p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}\Big)}{\overline{F}(p)+\overline{F}\Big(p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}\Big)}. \tag{30}$$

Recall that the consumer valuation is uniformly distributed on $[0,\overline{v}]$. Then $\theta=\frac{(1-\beta^{\frac{1}{\gamma}})(\overline{v}-p)-h}{2(1-\beta^{\frac{1}{\gamma}})(\overline{v}-p)-h}$ if $p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}\le\overline{v}$; and $\theta=0$ if $p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}>\overline{v}$. θ is an index which reflects the number of Type-2 consumers served in Period 1. The next lemma characterize some properties of θ.

LEMMA 1. *θ is increasing with respect to $\overline{v}$ and decreasing with respect to h, β, γ and p.*

Proof. if $p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}>\overline{v}$, the result is obviously true. Next we consider the case $p+\frac{h}{1-\beta^{\frac{1}{\gamma}}}\le\overline{v}$. By simple calculation, we have

$$\theta=1-\frac{\overline{v}-p}{2(\overline{v}-p)-X}=\frac{1}{2}-\frac{X}{4(\overline{v}-p)-2X}, \tag{31}$$

where $X=h/(1-\beta^{\frac{1}{\gamma}})$. Clearly, X is increasing in h, β and γ and θ is decreasing in X. Thus, θ is decreasing in h, β and γ. By the second Equation of (31), we can easily have that θ is increasing in $\overline{v}$ and decreasing in p. ■

Lemma 1 indicates that the number of Type-2 consumers served in Period 1 is larger when the consumers' desirability for the product becomes higher, or when consumers' holding cost, supply reliability, consumers' degree of risk aversion, or price becomes lower. This is intuitive since with lower holding cost, supply reliability or price, or with higher desirability or degree of risk aversion for the product, more consumers tend to purchase 2 units of products in Period 1.

Under capacity constraint (28), the retailer will certainly order $Q_1^* = K$ and $Q_2^* = 2N\overline{F}(p) - Q_1^*$. In this case, its expected profit is

$$\Pi^* = pK + \beta p(N\overline{F}(p) - K\theta), \tag{32}$$

where pK represents the profit of Period 1 and $\beta p(N\overline{F}(p) - K\theta)$ stands for the expected profit of Period 2.Obviously, the larger θ, the lower expected profit for the retailer. By Lemma 1, we know that the retailer will have a higher profit when h, β or γ are larger.

6.2 Fixed Quota Policy

Previously we have shown that with capacity constraint (28), the retailer's profit may suffer, as some portion of Period 2 demand are switched to Period 1, which leads to some demands of Period 1 not satisfied because of supply shortage. To alleviate such profit losses, it is often observed in practice that retailers impose fixed quota policy on consumers, e.g., allowing each consumer buy only one unit of product each time. In the following we study when it is beneficial for the retailer to impose fixed quota policy and when not.

Suppose that retailer implement fixed quota policy. If the retailer orders Q_{f1} units of products at the beginning of Period 1, then similar as above, it is optimal for the retailer to order $Q_{f2} = 2N\overline{F}(p) - Q_{f1}$ at the beginning of Period 2. Taking consideration of this equation, the retailer's expected profit can be written as a function of Q_{f1} (the subscript f stands for the case of fixed quota):

$$\begin{aligned} \Pi_f(Q_{f1}) &= pN\overline{F}(p) - H(Q_{f1} - N\overline{F}(p)) + \beta pN\overline{F}(p) + (1-\beta)p(Q_{f1} - N\overline{F}(p)) \\ &= (\beta+1)pN\overline{F}(p) + [(1-\beta)p - H](Q_{f1} - N\overline{F}(p)). \end{aligned} \tag{33}$$

The retailer should choose order quantity $Q_{f1} \in [N\overline{F}(p), K]$ to maximize its expected profit in (33). If $\beta > 1 - H/p$, then the optimal order quantity is $Q_{f1}^* = N\overline{F}(p)$ and the optimal profit is $\Pi_f^* = \Pi_f(Q_{f1}^*) = (1+\beta)pN\overline{F}(p)$. If $\beta \leq 1 - H/p$, then the optimal order quantity is $Q_{f1}^* = K$ and the optimal profit is $\Pi_f^* = \Pi_f(Q_{f1}^*) = (1+\beta)pN\overline{F}(p) + [(1-\beta)p - H](K - N\overline{F}(p))$. The next theorem identifies a threshold policy for the retailer on whether to implement fixed quota policy

THEOREM 5. *There exists a threshold $\delta_t > 1$ such that if $\delta \leq \delta_t$, then the retailer should implement fixed quota policy, and if $\delta > \delta_t$, then the retailer should not, where*

$$\delta_t = \begin{cases} \frac{1}{1-\beta\theta}, & if\ \beta > 1 - \frac{H}{p}; \\ \frac{H+\beta p}{\beta p(1-\theta)+H}, & if\ \beta \leq 1 - \frac{H}{p}. \end{cases} \tag{34}$$

Theorem 5 indicates that there exists a threshold such that when the capacity K is smaller the threshold, the retailer should implement fixed quota policy, and when the capacity K is larger than the threshold, it is optimal for the retailer not to implement fixed quota policy.

PROPOSITION 5. *δ_t is increasing with respect to $\overline{v}$, and decreasing with respect to γ, h and H.*

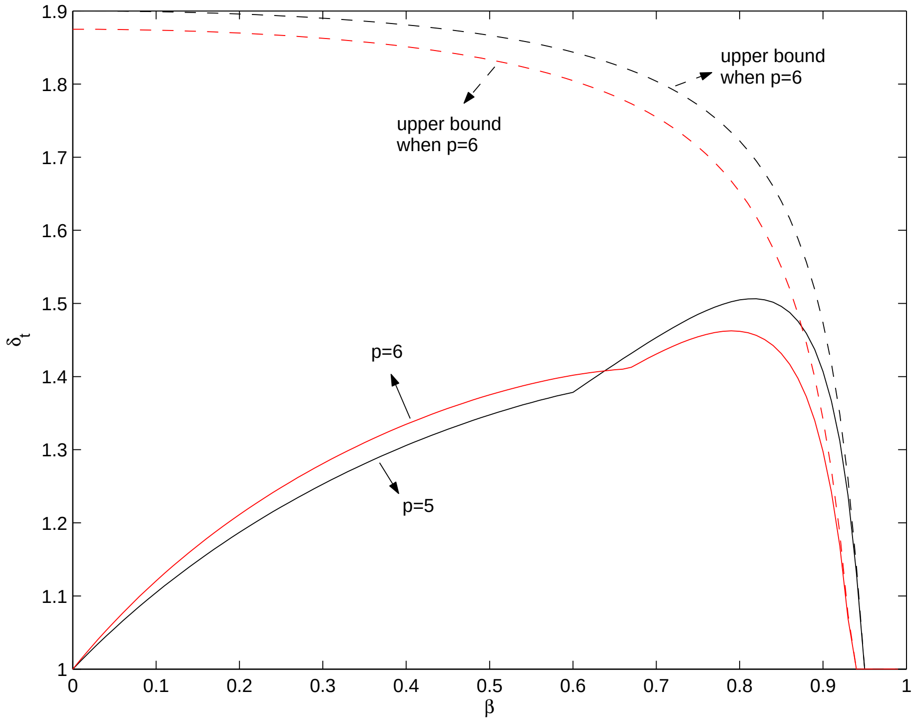
Proposition 5 implies that the fixed quota policy is more beneficial when consumers' desirability for the product is high, consumer holding cost is low, consumers' degree of risk aversion is high, or the retailer's holding cost is low. When consumers' desirability for the product becomes higher, consumer holding cost becomes lower, or consumers' degree of risk aversion becomes higher, there are more consumers purchasing two units of products in Period 1. As a result, there are more lost sales in Period 1 and less demand in Period 2. In this situation, fixed quota policy becomes more attractive. When the retailer's holding cost becomes lower, the retailer has a higher incentive to carry inventory if supply reliability is low. Thus, it is more attractive for the retailer to implement fixed quota policy to avoid lost sales in Period 1, and meanwhile carrying inventory to sell in Period 2.

How the attractiveness of the fixed quota policy changes with supply reliability and selling price is a bit more complicated. Next we provide two numerical examples to answer the question. The first example (Fig. 4), which is based on parameters: $H = 2$, $h = 0.5$, $\overline{v} = 10$, $\gamma = 0.5$, characterizes the relationship between δ_t and β (or p). The second example (Fig. 5), which is based on parameters: $H = 2$, $h = 0.5$, $\overline{v} = 10$, $\gamma = 0.5$, $N = 1000$ and $\delta = 1.3$, characterizes the relationship between $\Pi_f^* - \Pi^*$ and β (or p). From Figures 4 and 5, we can have the following observations:

First, the attractiveness of the fixed quota policy is first increasing then decreasing in supply reliability. In addition, both δ_t and $\Pi_f^* - \Pi^*$ reaches their maximum at $\beta = 1 - H/p$. These results are a bit counter-intuitive since one may think lower supply reliability would induce more Type-2 consumers, which makes fixed quota policy more attractive to the retailer. We explain the reasons in the following:

When $\beta \leq 1 - H/p$, the retailer will order $Q_{f1}^* = K$. In this case, by the fixed quota policy, the retailer can cut the lost sales in Period 1, hence increase its profit by $p[N\overline{F}(p) - (1-\theta)K]$. However, implementing fixed quota policy also incurs some costs: (i) Under the fixed quota policy, the retailer carries some extra inventories at the end of Period 1, in comparison with the case of no fixed quota policy. The cost is $H(K - N\overline{F}(p))$. (ii) To cut the lost sales, the fixed quota policy postpones the $K\theta$ consumers' requests for the second unit of product to Period 2. The postponement exposes $[N\overline{F}(p) - (1-\theta)K]$ consumers' demands to the risk of supply disruption (note that the retailer has $K - N\overline{F}(p)$ units of inventory at the beginning of Period 2). This lowers the retailer's expected profit by $(1-\beta)p[N\overline{F}(p) - (1-\theta)K]$. In summary, the net benefit of fixed

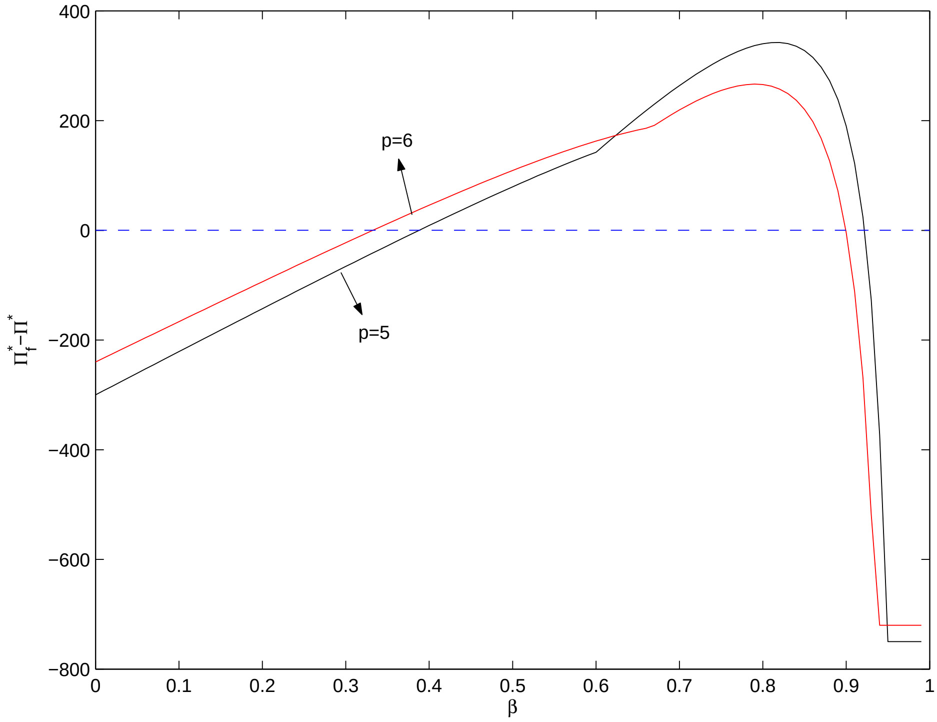
Figure 4 **The threshold δ_t v.s. supply reliability and price**



quota policy is $\beta p[N\overline{F}(p)-(1-\theta)K]-H(K-N\overline{F}(p))$. Note that the holding cost $H(K-N\overline{F}(p))$ is regardless of β, hence we only consider $\beta p[N\overline{F}(p)-(1-\theta)K]$. As supply reliability increases, although the quantity of lost sales $N\overline{F}(p)-(1-\theta)K$ decreases, β increases more significant than the quantity of lost sales decreases. This explains why the fixed quota policy is more attractive to the retailer as supply reliability increases when $\beta \le 1-H/p$.

When $\beta > 1-H/p$, it is optimal for the retailer to only order $N\overline{F}(p)$ before Period 1. In this case, on the one hand, the fixed quota policy increases the retailer's profit by $p[N\overline{F}(p)-(1-\theta)K]$ by cutting lost sales. On the other hand, implementing fixed quota policy postpone $K\theta$ consumers' requests for the second unit of product to Period 2, which makes these requests undergo the risk of supply disruption. Thus, the fixed quota policy also lowers the retailer's expected profit by $(1-\beta)K\theta$. In summary, the net benefit of the fixed quota policy in this case is $p[N\overline{F}(p)-(1-\theta)K]-(1-\beta)pK\theta = \beta p[N\overline{F}(p)-(1-\theta)K]-(1-\beta)p[K-N\overline{F}(p)]$. When β is not too big, as β increases, the quantity of lost sales $N\overline{F}(p)-(1-\theta)K$ decreases less significantly than β increases and the cost $(1-\beta)p[K-N\overline{F}(p)]$ decreases. Thus, $\beta p[N\overline{F}(p)-(1-\theta)K]-(1-\beta)p[K-N\overline{F}(p)]$ increases and the fixed quota policy is increasingly favored as supply reliability increases. When β is approaching to 1, the quantity of lost sales $(1-\beta)p[K-N\overline{F}(p)]$ decreases dramatically. This effects dominates the effects causing by the increasing of β and $-(1-\beta)p[K-N\overline{F}(p)]$. Therefore, the fixed quota policy becomes less attractive when β is sufficiently large and approaches to 1.

Figure 5 The net benefit of fixed quota policy v.s. supply reliability and price



Second, we observe that, the fixed quota policy is more attractive as selling price increases when supply reliability is low, but the opposite is true when supply reliability is high. This can be explained as follows. Increasing selling price has two effects: (a) decreasing total number of consumers who purchase products and decrease the number of Type-2 consumers, both of which decrease the lost sales in Period 1; (b) increasing the retailer's profit margin. When supply reliability is low, the net benefit of fixed quota policy is $\beta p[N\overline{F}(p)-(1-\theta)K]-H(K-N\overline{F}(p))$, and in this case Effect (a) is dominated by Effect (b). Thus, the fixed quota policy is increasingly favored as selling price increases when supply reliability is low. While under the case that supply reliability is high, the net benefit of the fixed quota policy is $\beta p[N\overline{F}(p)-(1-\theta)K]-(1-\beta)p[K-N\overline{F}(p)]$. In this case, as the selling price increases, both terms of the above benefit increase and counteract to each other. Hence, Effect (b) becomes less significant and dominated by Effect (a). Then the attractiveness of the fixed quota policy decreases as the selling price increases when supply reliability is high.

7. Extensions

In this section, we provide some extensions to the basic models. The first extension allows the retailer to adjust price in Period 2 when supply disruption occurs (we refer to this as *Responsive Pricing*). In the second extension, we study similar problems in a more general setting: the retailer's received quantity in Period 2 is proportional to the order quantity, where the obtained proportion is a random variable over [0,1] (we refer to this as *Random Yield*).

7.1 Responsive pricing

In this subsection, we discuss the case that the retailer can adjust its selling price during Period 2 if supply disruption occurs. Let p_r be the selling price of Period 2 under supply disruption and η be a fulfill rate that measures the probability of a consumer getting his demand from the retailer's inventory when supply disruption occurs. We assume $p_r \geq p$, i.e., the price under disruption is no less than the regular price. For tractability, we focus on risk neutral consumers in this subsection, i.e., $U(x) = x$. Other parameters are the same as in Section 3 and the sequence of events is also the same as in Section 3 except that the retailer will switch its selling price from p to p_r in the case of supply disruption at the beginning of Period 2.

Similar to the previous analysis, we adopt the rational expectation (RE) equilibrium concept. In particular, we assume that all the consumers have rational expectation on both p_r and η. To conduct equilibrium analysis, we have to characterize the consumers' purchasing behavior under given beliefs of the consumers over p_r and η. Denote these belief by $p_{r,c}$ and η_c. Consider a consumer with valuation v. Let z_{r1} and z_{r2} be the consumer's purchasing quantities of Period 1 and Period 2 respectively. Clearly, if $v \leq p$, then $z_{r1} = z_{r2} = 0$. If $v \in (p, p_{r,c})$, then $z_{r1} \in \{1, 2\}$ and $z_{r2} = 0$. If $v \geq p_{r,c}$, then $z_{r1} \in \{1, 2\}$ and $z_{r2} = 2 - z_{r1}$. This means when one consumer's valuation, beliefs and purchasing quantity in Period 1 z_{r1} are determined, the purchasing quantity in Period 2 z_{r2} is also determined. Thus, let $\pi_r(z_{r1}, p_{r,c}, \eta_c)$ be a consumer's expected profit with purchasing quantity z_{r1} and beliefs $p_{r,c}$, η_c. When $v > p$, we have

$$\begin{aligned} \pi_r(1, p_{r,c}, \eta_c) &= (1+\beta)(v-p) + (1-\beta)\eta_c(v - p_{r,c})^+, \\ \pi_r(2, p_{r,c}, \eta_c) &= 2(v-p) - h. \end{aligned}$$

respectively. Thus, we have the following theorem.

THEOREM 6. *For any given $p_{r,c} \geq p$ and $0 \leq \eta_c \leq 1$, the optimal purchasing policy of a consumer with valuation v is as follows:*

(1) If $v \leq p$, $z^*_{r1}(\eta_c, p_{r,c}) = z^*_{r2}(\eta_c, p_{r,c}) = 0$

(2) If $v \in (p, p_{r,c})$, then

(i) When $v \geq p + h/(1-\beta)$, $z^*_{r1}(\eta_c, p_{r,c}) = 2$, $\quad z^*_{r2}(\eta_c, p_{r,c}) = 0$;

(ii) When $v < p + h/(1-\beta)$, $z^*_{r1}(\eta_c, p_{r,c}) = 1$, $\quad z^*_{r2}(\eta_c, p_{r,c}) = 0$.

(3) If $v \geq p_r$, then

(i) When $v \geq T_r(\eta, p_{r,c})$, $z^*_{r1}(\eta_c, p_{r,c}) = 2$, $\quad z^*_{r2}(\eta_c, p_{r,c}) = 0$,

(ii) When $v < T_r(\eta, p_{r,c})$, $z^*_{r1}(\eta_c, p_{r,c}) = 1$, $\quad z^*_{r2}(\eta_c, p_{r,c}) = 1$,

where

$$T_r(\eta, p_{r,c}) = \frac{(1-\beta)(p - \eta p_r) + h}{(1-\beta)(1-\eta)}. \tag{35}$$

Theoretically, $p_{r,c}$ can be any value that no less than p. However, if $p_{r,c} \geq h/(1-\beta)$, then it is easy to know

$$p_{r,c} \geq \frac{h}{1-\beta} \geq T_r(\eta_c, p_{r,c}).$$

Thus, by Parts (2) and (3) of Theorem 6, no consumers will buy any products in Period 2, which is equivalent to the case that $\eta_c = 0$. Therefore, we only consider the case that $p_{r,c} \in \Big[p, p + h/(1-\beta)\Big)$ hereafter. In this case, it always hold that

$$T_r(\eta_c, p_{r,c}) \geq \frac{h}{1-\beta} \geq p_{r,c}.$$

Next we consider the retailer's decisions, i.e., the order quantities of Period 1 and 2, Q_{r1}, Q_{r2}, and the price of Period 2 under supply disruption p_r. For given η_c and $p_{r,c}$, the retailer's problem is to choose proper Q_{r1}, Q_{r2} and p_r to maximize its expected profit

$$\begin{aligned}
\max_{Q_{r1}, Q_{r2}, p_r} \quad & \Pi_r(Q_{r1}, Q_{r2}, p_r, \eta_c, p_{r,c}) \\
&= \min\Big\{Q_{r1},\ N\overline{F}(p) + N\overline{F}\big(T_r(\eta_c, p_{r,c})\big)\Big\} - H\Big[Q_{r1} - N\overline{F}(p) - N\overline{F}\big(T(\eta_c, p_{r,c})\big)\Big]^+ \\
&+ \beta p \min\Big\{Q_{r2} + \Big[Q_{r1} - N\overline{F}(p) - N\overline{F}\big(T(\eta_c, p_{r,c})\big)\Big]^+,\ NF\big(T_r(\eta_c, p_{r,c})\big) - NF(p_{r,c})\Big\} \\
&+ (1-\beta) p_{r,c} \Big[Q_1 - N\overline{F}(p) - N\overline{F}(T(\eta_c, p_{r,c}))\Big]^+
\end{aligned} \tag{36}$$

In Eqn. (36), the first term is the retailer's expected revenue in Period 1, the second term stands for the holding cost, the third term is the expected revenue with successful supply and the last term is the expected profit under supply disruption. For given η_c and $p_{r,c}$, denote the retailer's optimal decisions that maximizes Problem (36) by $Q^*_{r1}(\eta_c, p_{r,c})$, $Q^*_{r2}(\eta_c, p_{r,c})$ and $p^*_r(\eta_c, p_{r,c})$. Then we introduce the definition of RE equilibrium under the case of responsive pricing.

Definition 2. A RE equilibrium under the case of responsive pricing satisfies the following conditions:

(i) For given beliefs η_c and $p_{r,c}$, the optimal purchasing policy of any consumer who possesses valuation v is $z_1 = z^*_1(\eta_c, p_{r,c})$, $z_2 = z^*_2(\eta_c, p_{r,c})$, where $z^*_1(\eta_c, p_{r,c})$ and $z^*_2(\eta_c, p_{r,c})$ are defined in Theorem 6.

(ii) For given belief η_c and $p_{r,c}$, the retailer's optimal order quantities and responsive price satisfy $Q_1 = Q^*_1(\eta_c, p_{r,c})$, $Q_2 = Q^*_2(\eta_c, p_{r,c})$ and $p_r = p^*_r(\eta_c, p_{r,c})$.

(iii) The beliefs of all the consumers η_c and $p_{r,c}$ are consistent with the realized ones (η and $p_{r,c}$), i.e.,

$$\eta_c = \eta \triangleq \min\left\{\frac{\Big[Q^*_{r1}(\eta_c, p_{r,c}) - N\overline{F}(p) - N\overline{F}\big(T_r(\eta_c, p_{r,c})\big)\Big]^+}{N\Big[F\big(T_r(\eta_c, p_{r,c})\big) - F(p_{r,c})\Big]},\ 1\right\}, \tag{37}$$

$$p_{r,c} = p^*_r(\eta_c, p_{r,c}) \tag{38}$$

In Eqn. (37), $\Big[Q^*_{r1}(\eta_c,p_{r,c})-N\overline{F}(p)-N\overline{F}\big(T_r(\eta_c,p_{r,c})\big)\Big]^+$ stands for the retailer's inventory at the beginning of Period 2 and $N\Big[F\big(T_r(\eta_c,p_{r,c})\big)-F(p_{r,c})\Big]$ is the number of consumers who will purchase products in Period 2.

Next we conduct equilibrium analysis. Since in RE equilibrium, η_c and $p_{r,c}$ must be consistent with η and p_r. Thus, we use η and p_r to replace η_c and $p_{r,c}$ hereafter. By similar arguments as before Eqn. (8), we have that the retailer will not choose $Q_{1r}<N\overline{F}(p)+N\overline{F}\big(T_r(\eta,p_r)\big)$ or $Q_{1r}>N\overline{F}(p)+N\overline{F}(p_r)$. Thus, $Q_{1r}\in\Big[N\overline{F}(p)+N\overline{F}\big(T_r(\eta,p_r)\big),\ N\overline{F}(p)+N\overline{F}(p_r)\Big]$. Submitting (37) and (38) into (36), we have that the retailer's optimal problem is

$$\begin{aligned}\max_{\eta,p_r}\ \Pi_r\ &=\ 2Np\overline{F}(p)-(1-\beta)pN\Big[F\big(T_r(\eta,p_r)\big)-F(p)\Big]\\ &+\ N\cdot\Big[(1-\beta)\eta p_r-\eta H\Big]\cdot\Big[F\big(T_r(\eta,p_r)\big)-F(p_r)\Big] \qquad (39)\\ &s.t.\ \ \eta\in[0,1],\ p_r\in\Big[p,p+\frac{h}{1-\beta}\Big). \qquad (40)\end{aligned}$$

Once the optimal η, p_r are obtained, we can easily calculate the optimal Q from Eqn. (37). In the rest of this subsection, we transfer the decision of Q and p_r to η and p_r.

To facilitate analysis, we concentrate on a special case:

$$F(v)=\begin{cases}v/\overline{v}, & if\ 0\le v\le\overline{v},\\ 1, & if\ v>\overline{v}.\end{cases} \qquad (41)$$

Let $\overline{\eta}(p_r)=(\overline{v}-p-\frac{h}{1-\beta})/(\overline{v}-p_r)$, which satisfies $T_r(\overline{\eta}(p_r),p_r)=\overline{v}$. By Eqn. (41), Problem (39) can be split into the following three subproblems:

$$\max_{\eta,p_r}\ \Pi_{r1}(\eta,p_r)=\frac{2Np(\overline{v}-p)}{\overline{v}}+\frac{N}{\overline{v}}\Big\{\Big(p_r-\frac{H}{1-\beta}\Big)\frac{\eta[h-(1-\beta)(p_r-p)]}{1-\eta}-\frac{p[h-(1-\beta)\eta(p_r-p)]}{1-\eta}\Big\} \qquad (42)$$

$$s.t.\ \ 0\le\eta\le\max\{0,\overline{\eta}(p_r)\},\ p\le p_r<\min\Big\{\overline{v},p+\frac{h}{1-\beta}\Big\}.$$

$$\max_{\eta,p_r}\ \Pi_{r2}(\eta,p_r)=\frac{2Np(\overline{v}-p)}{\overline{v}}+\frac{N}{\overline{v}}\Big[\eta(1-\beta)\Big(p_r-\frac{H}{1-\beta}\Big)(\overline{v}-p_r)-(1-\beta)(\overline{v}-p)p\Big] \qquad (43)$$

$$s.t.\ \ \max\{0,\overline{\eta}(p_r)\}\le\eta\le1,\ p\le p_r<\min\Big\{\overline{v},p+\frac{h}{1-\beta}\Big\}.$$

$$\max_{\eta,p_r}\ \Pi_{r3}(\eta,p_r)=\frac{2Np(\overline{v}-p)}{\overline{v}}-\frac{N}{\overline{v}}\Big\{(1-\beta)(\overline{v}-p)p\Big\} \qquad (44)$$

$$s.t.\ \ 0\le\eta\le1,\ \min\Big\{\overline{v},p+\frac{h}{1-\beta}\Big\}\le p_r<p+\frac{h}{1-\beta}.$$

THEOREM 7. *(1) When $H>h+p(1-\beta)$, then $\eta^*=0$ and p^*_r can be any value in $\big[p,\overline{v}\big)$;*
(2) When $h<H\le h+p(1-\beta)$, then there exists a threshold $v_{r,t}(\beta)\in\Big[\max\big\{p+\frac{h}{1-\beta},\ \frac{p(H+h)}{H}\big\},\ 2p+\frac{2h}{1-\beta}-\frac{H}{1-\beta}\Big]$ such that
(i) if $\overline{v}\le\frac{H}{1-\beta}$, $\eta^*=0$, p^*_r can be any value in $[p,\overline{v})$;

(ii) if $\frac{H}{1-\beta} < \overline{v} < v_{r,t}(\beta)$*, then* $\eta^* = 1$ *and*

$$p_r^* = \underset{p \le p_r \le \min\{\overline{v}, p+\frac{h}{1-\beta}-\epsilon\}}{\arg\max} \Big(p_r - \frac{H}{1-\beta}\Big)(\overline{v} - p_r),$$

where ϵ *is a very small positive number.*

(iii) if $\overline{v} \ge v_{r,t}(\beta)$*,* $\eta^* = 0$*,* p_r^* *can be any value in* $[p, p+\frac{h}{1-\beta})$*;*

(3) When $H \le h$*, then*

(i) if $\overline{v} \le \frac{H}{1-\beta}$*,* $\eta^* = 0$*,* p_r^* *can be any value in* $\big[p, \overline{v}\big)$*;*

(ii) if $\overline{v} > \frac{H}{1-\beta}$*, then* $\eta^* = 1$ *and*

$$p_r^* = \underset{p \le p_r \le \min\{\overline{v}, p+\frac{h}{1-\beta}-\epsilon\}}{\arg\max} \Big(p_r - \frac{H}{1-\beta}\Big)(\overline{v} - p_r),$$

where ϵ *is a very small positive number.*

Proof. Part (1). If $\overline{v} < p + \frac{h}{1-\beta} < \frac{H}{1-\beta}$, then $\overline{\eta}(p_r) < 0$. Thus, Problem (39) reduces to Subproblems (43) and (44). By observation, the optimal value of Subproblem (44) is included in Subproblem (43) with $\eta = 0$. Therefore, we only need to consider Subproblem (43). It is easy to know that $(p_r - \frac{H}{1-\beta})(\overline{v} - p_r) < 0$, which indicates that the optimal solution of Problem (39) is $\eta^* = 0$, p_r^* can be any value in $[p, \overline{v})$.

If $p + \frac{h}{1-\beta} \le \overline{v} < \frac{H}{1-\beta}$, then $\overline{\eta}(p_r) \ge 0$. Then Problem (39) reduces to Subproblems (42) and (43). First, let's consider Subproblems (42), which is equivalent to

$$\max_{\eta, p_r} \ \Pi_{r1}(\eta, p_r) = \frac{2Np(\overline{v} - p)}{\overline{v}} - \frac{N}{\overline{v}}\left[(1-\beta)\Big(p_r - p - \frac{H}{1-\beta}\Big)\Big(p_r - p - \frac{h}{1-\beta}\Big)\frac{\eta}{1-\eta} + ph\right]$$

$$s.t.\ \ 0 \le \eta \le \overline{\eta}(p_r),\ p \le p_r < p + \frac{h}{1-\beta}.$$

It is easy to know that $(p_r - p - \frac{H}{1-\beta})(p_r - p - \frac{h}{1-\beta}) > 0$. Thus, the optimal solution to Subproblem (42) is $\eta_1^* = 0$, p_{r1}^* can be any value in $[p, h + \frac{h}{1-\beta})$. Second, we consider Subproblem (43). Since $v < \frac{H}{1-\beta}$, then $(p_r - \frac{H}{1-\beta})(\overline{v} - p_r) < 0$. Thus, for any fixed $p_r \in [p, p + \frac{h}{1-\beta})$, the optimal η is $\eta_2^* = \overline{\eta}(p_r)$. Finally, by the continuity of $\widetilde{\Pi}(\eta, p_r)$ in (39), we have $\Pi_{r1}(\eta_1^*, p_{r1}^*) \ge \Pi_{r1}(\widetilde{\eta}(p_r), p_r) = \Pi_{r2}(\eta_2^*(p_r), p_r)$, which indicates that the optimal solution to Problem (39) is $\eta^* = 0$ and p_r^* can be any value in $[p, h + \frac{h}{1-\beta})$.

If $\overline{v} \ge \frac{H}{1-\beta} > p + \frac{h}{1-\beta}$, then $\overline{\eta}(p_r) > 0$. Thus Problem (39) reduces to Subproblems (42) and (43). With a similar argument as above, the optimal solution to Subproblem (42) is $\eta_1^* = 0$, $p_{r1}^* \in [p, h + \frac{h}{1-\beta})$. Second, let's consider Subproblem (43). For any fixed $p_r \in [p, p + \frac{h}{1-\beta})$, it is optimal to choose

$$\eta_2^*(p_r) = \begin{cases} \overline{\eta}(p_r), & if\ p_r \le \frac{H}{1-\beta}, \\ 1, & if\ p_r > \frac{H}{1-\beta}. \end{cases}$$

In addition, we have $\Pi_{r2}(\eta_2^*(p_1),p_1) < \Pi_{r2}(\eta_2^*(p_2),p_2)$ for any $p_1 \le \frac{H}{1-\beta}$ and $p_2 > \frac{H}{1-\beta}$. Thus, the optimal solution to Subproblem (43) is $\eta_2^*=1$, $p_{r2}^* = \underset{p\le p_r\le p+\frac{h}{1-\beta}-\epsilon}{\arg\max}(p_r - \frac{H}{1-\beta})(\overline{v}-p_r)$, where ϵ is a very small positive number. Next, we compare $\Pi_{r1}(\eta_1^*,p_{r1}^*)$ and $\Pi_{r2}(\eta_2^*,p_{r2}^*)$. It is easy to obtain

$$\Pi_{r1}(\eta_1^*,p_{r1}^*) = \frac{2Np(\overline{v}-p)}{\overline{v}} - \frac{Nph}{\overline{v}}, \tag{45}$$

$$\Pi_{r2}(\eta_2^*,p_{r2}^*) = \frac{2Np(\overline{v}-p)}{\overline{v}} + \frac{N}{\overline{v}}J(\overline{v}), \tag{46}$$

where $J(\overline{v}) = (1-\beta)\cdot\max_{p_r}\{(p_r-\frac{H}{1-\beta})(\overline{v}-p_r)-(\overline{v}-p)p : p\le p_r\le p+\frac{h}{1-\beta}-\epsilon\}$. According to Part (i) of Lemma 2, which is immediately below this theorem, $J(\overline{v})$ is a decreasing function when $H>h$. In addition, $J(\frac{H}{1-\beta}) = -(H-(1-\beta)p)p < -ph$, which implies $J(\overline{v}) < -ph$ for any $\overline{v}\ge\frac{H}{1-\beta}$. Therefore, $\Pi_{r1}(\eta_1^*,p_{r1}^*) > \Pi_{r2}(\eta_2^*,p_{r2}^*)$, which means that $\eta^*=\eta_1^*=0$ $p_d^*=p_{r1}^*$ can be any value in $[p, h+\frac{h}{1-\beta})$.

Part (2). If $\overline{v}\le\frac{H}{1-\beta}\le p+\frac{h}{1-\beta}$, Problem (39) reduces to Subproblems (43) and (44). By a similar argument with Part (1), we can have $\eta^*=0$, p_r^* can be any value in $[p,\overline{v})$.

If $\frac{H}{1-\beta}<\overline{v}\le p+\frac{h}{1-\beta}$, Problem (39) reduces to Subproblems (43) and (44). Similarly, we have $\eta^*=1$, $p_r^*=\underset{p\le p_r\le\overline{v}}{\arg\max}(p_r-\frac{H}{1-\beta})(\overline{v}-p_r)$.

If $\overline{v}>p+\frac{h}{1-\beta}\ge\frac{H}{1-\beta}$, Problem (39) reduces to Subproblems (42) and (43). With a similar argument as Part (1), we have the optimal solution to Subproblem (42) is $\eta_1^*=0$,$p_{r1}^*\in\big[p,p+\frac{h}{1-\beta}\big)$, and the optimal solution to Subproblem (43) is $\eta_2^*=1$, $p_{r2}^*=\underset{p\le p_r\le p+\frac{h}{1-\beta}-\epsilon}{\arg\max}(p_r-\frac{H}{1-\beta})(\overline{v}-p_r)$, where ϵ is a very small positive number. Finally, we compare $\Pi_{r1}(\eta_1^*,p_{r1}^*)$ and $\Pi_{r2}(\eta_2^*,p_{r2}^*)$, which are also equal to (45) and (46) respectively. Recall that $J(\overline{v})$ is a decreasing function. According to

$$J\Big(p+\frac{h}{1-\beta}\Big) = -ph + \frac{((1-\beta)p+h-H)^2}{4(1-\beta)} \ge -ph,$$

$$J\Big(2p+\frac{2h}{1-\beta}-\frac{H}{1-\beta}\Big) = \Big(p+\frac{h}{1-\beta}-\frac{H}{1-\beta}\Big)(h-H-\beta p)-ph \le -ph,$$

there exists a threshold $v_{r,t}(\beta)\in[p+\frac{h}{1-\beta}, 2p+\frac{2h}{1-\beta}-\frac{H}{1-\beta}]$ such that if $\overline{v}<v_{r,t}(\beta)$, then $\eta^*=1$, and if $\overline{v}\le v_{r,t}(\beta)$, then $\eta^*=1$. Furthermore, by Part (ii) of Lemma **??**, we also have $v_{r,t}(\beta)\ge p(H+h)/H$. Thus, $v_{r,t}(\beta)\in\Big[\max\{p+\frac{h}{1-\beta},\frac{p(H+h)}{H}\},\ 2p+\frac{2h}{1-\beta}-\frac{H}{1-\beta}\Big]$.

Part (3). It is easy for one to prove that following results with a similar method as above:

(a) if $\overline{v}\le\frac{H}{1-\beta}\le p+\frac{h}{1-\beta}$, $\eta^*=0$ and p_d^* can be any value in $[p,\overline{v})$;

(b) if $\frac{H}{1-\beta}<\overline{v}\le p+\frac{h}{1-\beta}$, $\eta^*=1$ and $p_r^*=\underset{p\le p_r\le\overline{v}}{\arg\max}(p_r-\frac{H}{1-\beta})(\overline{v}-p_r)$.

Next, we consider the case $\overline{v}>p+\frac{h}{1-\beta}\ge\frac{H}{1-\beta}$. In this case, Problem (39) reduces to Subproblems (42) and (43). For Subproblem (43), by a similar argument with Part (1), we have the optimal solution is $\eta_2^*=1$, $p_{r2}^*=\underset{p\le p_r\le p+\frac{h}{1-\beta}-\epsilon}{\arg\max}(p_r-\frac{H}{1-\beta})(\overline{v}-p_r)$, where ϵ is a very small positive number. For Subproblem (42), since $H\le h$, then for any given $p_r\in[p,p+\frac{h}{1-\beta})$, it is optimal to choose

$$\eta_1^*(p_r)=\begin{cases}0, & if\ p_r\le p+\frac{H}{1-\beta},\\ \overline{\eta}(p_r), & if\ p_r>p+\frac{H}{1-\beta}.\end{cases}$$

Clearly, $\Pi_{r1}(\eta_1^*(p_1),p_1)<\Pi_{r1}(\eta_1^*(p_2),p_2)$ for any $p_1\le p+\frac{H}{1-\beta}$ and $p_2>p+\frac{H}{1-\beta}$. Therefore, the optimal p_r to Subproblem (42) lies between $p+\frac{H}{1-\beta}$ and $p+\frac{h}{1-\beta}$ (we call it p_{r1}^*). Furthermore, we have $\eta_1^*=\overline{\eta}(p_{r1}^*)$. Comparing the optimal values of this two subproblems, we have

$$\Pi_{r2}(\eta_2^*,p_{r2}^*)\ge\Pi_{r2}(\overline{\eta}(p_{r1}^*),p_{r1}^*)=\Pi_{r1}(\eta_1^*,p_{r1}^*)$$

by continuity of Π_r in (39) with respect to (η,p_r). Therefore, in this case, $\eta^*=\eta_2^*=1$ and $p_r^*=p_{r2}^*=\underset{p\le p_r\le p+\frac{h}{1-\beta}-\epsilon}{\arg\max}(p_r-\frac{H}{1-\beta})(\overline{v}-p_r)$. ■

LEMMA 2. *Suppose $H>h$. Let $J(\overline{v})=(1-\beta)\cdot\max_{p_r}\{(p_r-\frac{H}{1-\beta})(\overline{v}-p_r)-(\overline{v}-p)p:p\le p_r\le p+\frac{h}{1-\beta}-\epsilon\}$.*

(i) $J(\overline{v})$ is a decreasing function.

(ii) let $v_{r,t}(\beta)$ be the solution of $J(\overline{v})=-ph$. If $\beta\le1-\frac{H^2}{p(H-h)}$, then $v_{r,t}(\beta)=\frac{p(H+h)}{H}$. If $\beta>1-\frac{H^2}{p(H-h)}$, then $v_{r,t}(\beta)$ satisfies

$$\frac{1}{4}\Big(v_{r,t}(\beta)-\frac{H}{1-\beta}\Big)^2(1-\beta)-(v_{r,t}(\beta)-p)p(1-\beta)=-ph,\tag{47}$$

and $v_{r,t}(\beta)>\frac{p(H+h)}{H}$

Proof. Part (i). Let $K(v,p_r)=(p_r-\frac{H}{1-\beta})(v-p_r)-(v-p)p$. It is easy to obtain that for any $p_r\in[p,p+\frac{h}{1-\beta})$,

$$\frac{\partial K}{\partial v}=p_r-\frac{H}{1-\beta}-p<0,$$

which implies that $K(v,p_r)$ is increasing in v. For any $v_1<v_2$, let $J(v_1)=(1-\beta)K(v_1,p_{r1})$ and $J(v_2)=(1-\beta)K(v_2,p_{r2})$. Since $K(v_1,p_{r1})\ge K(v_1,p_{r2})\ge K(v_2,p_{r2})$, then $J(v_1)\ge J(v_2)$.

Part (ii). It is easy to have that

$$J(\overline{v})=\begin{cases}-(\overline{v}-p)H, & if\ \overline{v}\le 2p-\frac{H}{1-\beta},\\ \frac{1}{4}\Big(\overline{v}-\frac{H}{1-\beta}\Big)^2(1-\beta)-(\overline{v}-p)p(1-\beta), & if\ 2p-\frac{H}{1-\beta}<\overline{v}\le 2p+\frac{2h}{1-\beta}-\frac{H}{1-\beta},\\ -ph-(H-h)\Big(\overline{v}-p-\frac{h}{1-\beta}\Big), & if\ \overline{v}>2p+\frac{2h}{1-\beta}-\frac{H}{1-\beta}.\end{cases}\tag{48}$$

If $\beta\le1-\frac{H^2}{p(H-h)}$, then

$$\frac{p(H+h)}{H}\le 2p-\frac{H}{1-\beta}.\tag{49}$$

In addition, we have $J\left(\frac{p(H+h)}{H}\right) = -ph$. Thus, $v_{r,t}(\beta) = \frac{p(H+h)}{H}$ in this case.

If $\beta > 1 - \frac{H^2}{p(H-h)}$, then it can be easily verifies that $J(\overline{v}) > -ph$ for all $\overline{v} \leq 2p - \frac{H}{1-\beta}$. Thus, we have $v_{r,t}(\beta) > 2p - \frac{H}{1-\beta}$ since $J(\overline{v})$ is a decreasing function. By Eqn. (48), we have Eqn. (47) is true. Next we prove $v_{r,t}(\beta) \geq \frac{p(H+h)}{H}$. Consider two functions: $J_1(\overline{v}) = -(\overline{v}-p)H$ and $J_2(\overline{v}) = \frac{1}{4}\left(\overline{v} - \frac{H}{1-\beta}\right)^2(1-\beta) - (\overline{v}-p)p(1-\beta)$. Then we have $J_1'(\overline{v}) = -H$ and $J_2'(\overline{v}) = \frac{1}{2}(1-\beta)\left(\overline{v} - \frac{H}{1-\beta} - 2p\right)$. Since $J_2'(\overline{v}) > J_1'(\overline{v}),\ \forall \overline{v} > 2p - \frac{H}{1-\beta}$ and $J_1\left(2p - \frac{H}{1-\beta}\right) = J_2\left(2p - \frac{H}{1-\beta}\right)$, then we have $J_2(\overline{v}) \geq J_1(\overline{v}),\ \forall \overline{v} > 2p - \frac{H}{1-\beta}$, which, together with $J\left(\frac{p(H+h)}{H}\right) = -ph$ and $v_{r,t}(\beta)$ satisfies $J_2(v_{r,t}(\beta)) = 0$, implies $v_{r,t}(\beta) \geq \frac{p(H+h)}{H}$. ■

Let Π_r^* and Π^* be the retailer's optimal expected profit with responsive pricing and without responsive pricing respectively. By comparing Theorems 2 and 7, we can show that $\Pi_r^* \geq \Pi^*$, which is intuitive. A more interesting question is: in what situation the responsive pricing can strictly increase the retailer's expected profit, compared with the case of no responsive pricing. The following proposition answers the question.

PROPOSITION 6. *(i) When* $H > h$, $\Pi_r^* > \Pi^*$ *for* $(\beta,\overline{v}) \in \left\{(\beta,\overline{v}) \mid \max\{2p - \frac{H}{1-\beta}, \frac{H}{1-\beta}\} < v_{r,t}(\beta),\ 1 - \frac{H^2}{p(H-h)} < \beta < 1 - \frac{H}{p}\right\}$ *and* $\Pi_r^* = \Pi^*$ *else.*

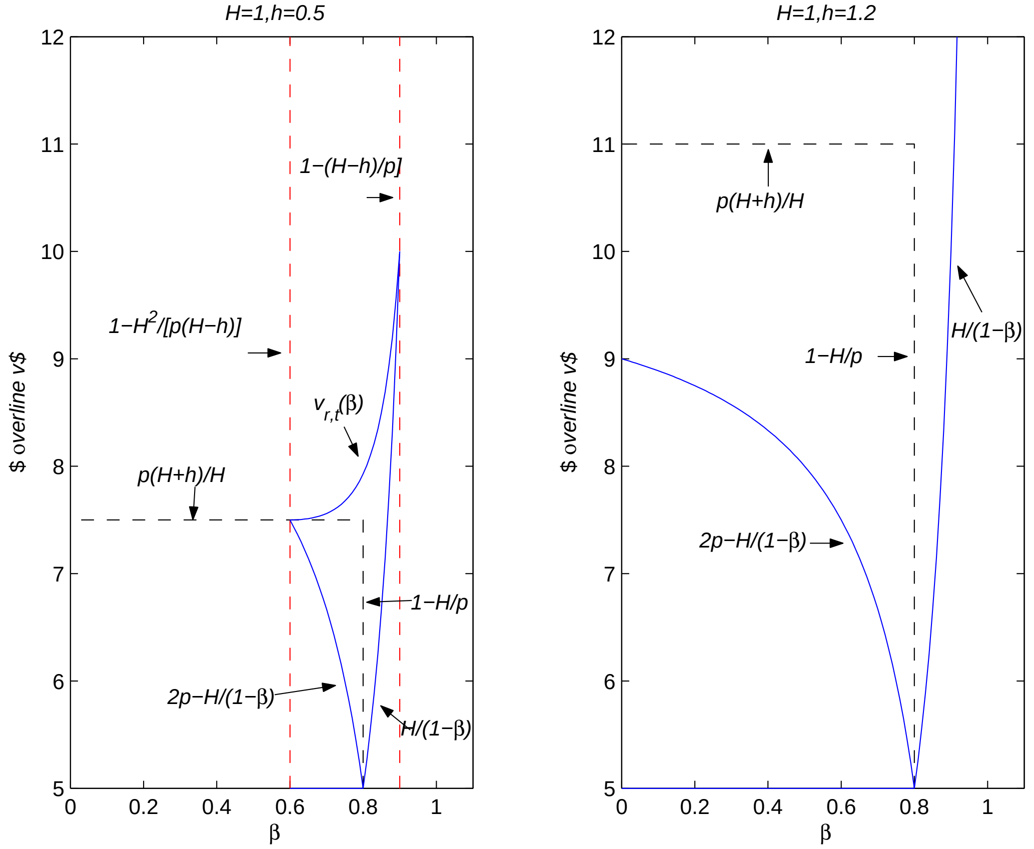
(ii) When $H \leq h$, $\Pi_r^* > \Pi^*$ *for* $(\beta,\overline{v}) \in \left\{(\beta,\overline{v}) \mid \overline{v} > \max\{2p - \frac{H}{1-\beta}, \frac{H}{1-\beta}\}\right\}$ *and* $\Pi_r^* = \Pi^*$ *otherwise.*

Proposition 6 shows that if the retailer's holding cost rate is higher than the consumers', the responsive pricing can increase the retailer's profit when both the consumers' desirability for the product and the supply reliability is median, while if the retailer's holding cost rate is not higher than the consumers', then the responsive pricing can increase the retailer's profit when the consumers' desirability is relatively high. To show this more clearly, we provide a numerical example, in which $p = 5$, $H = 1$. The results is demonstrated in Fig. 6. In the left side, where $h = 0.5 < H$, the benefit of responsive pricing is positive when $(\beta,\overline{v})$ falls in the region that is formed by the three solid lines. In the right side, where $h = 1.2 > H$, the benefit of responsive pricing is positive when $(\beta,\overline{v})$ is above the two solid lines. From this numerical example, we can observe that responsive pricing is more effective for the retailer when the its holding cost rate is lower than the consumers', in comparison with the case that the retailer's holding cost rate is higher than the consumers'.

7.2 Random proportional yield

So far our analysis has been based on disruption model. In this section, we generalize our model to the case of random yield model. Assume that at the beginning of Period 2 the retailer can get only xQ_2 units of products if it orders Q_2 units, where x is a random variable with support $[0,1]$ and cumulative distribution function $B(x)$. Here x can be either a continuous random variable or a discrete random variable. Obviously, the case of supply disruption discussed in the previous sections is a special case here, when x is a binary random variable with probability β being 1 and

Figure 6   The situations where the benefit of responsive pricing is positive



with probability $1-\beta$ being 0. Here the retailer's purchasing cost c is critical and we can not normalize it to 0 as before. We assume that the retailer only need to pay the purchasing cost to what it received. Let $\mu = E(x)$. Then the retailer's problem is deciding order quantities Q_1 and Q_2 for Periods 1 and 2 respectively to maximize its expected profit, i.e.,

$$\max_{Q_1,Q_2} \ \Pi(Q_1,Q_2) = (p-c)Q_1 - H\left[Q_1 - N\overline{F}(p) - N\overline{F}\left(p+\frac{h}{1-\alpha^{\frac{1}{\gamma}}}\right)\right] + pE\ \min\{xQ_2, 2N\overline{F}(p)-Q_1\} - c\mu Q_2 \tag{50}$$

$$s.t.\ \alpha = E\ \min\left\{1, \frac{xQ_2 + Q_1 - N\overline{F}(p) - N\overline{F}\left(p+\frac{h}{1-\alpha^{\frac{1}{\gamma}}}\right)}{NF\left(p+\frac{h}{1-\alpha^{\frac{1}{\gamma}}}\right) - NF(p)}\right\}, \tag{51}$$

$$Q_1 \in \left[N\overline{F}(p) + N\overline{F}\left(p+\frac{h}{1-\alpha^{\frac{1}{\gamma}}}\right), 2N\overline{F}(p)\right]. \tag{52}$$

Next we solve the above problem. First, we solve the optimal Q_2 for any given $Q_1 \in [N\overline{F}(p) + N\overline{F}(p+\frac{h}{1-\alpha^{\frac{1}{\gamma}}}), 2N\overline{F}(p)]$. By observation, we only need choose Q_2 to maximize

$$\widetilde{\Pi} = pE\ \min\{xQ_2, 2N\overline{F}(p)-Q\} - c\mu Q_2 = p\overline{B}\left(\frac{2N\overline{F}(p)-Q_1}{Q_2}\right)[2N\overline{F}(p)-Q_1] + \left[p\int_0^{\frac{2N\overline{F}(p)-Q_1}{Q_2}} x dB(x) - c\mu\right]Q_2. \tag{53}$$

It can be easily verified that $\widetilde{\Pi}$ is concave function of Q_2 and the optimal order quantity of at the beginning of Period 2, Q_2^*, satisfies

$$p\int_0^{\frac{2N\overline{F}(p)-Q_1}{Q_2^*}} xdB(x) = c\mu \tag{54}$$

Let $\xi^* = \frac{[2N\overline{F}(p)-Q_1]}{Q_2^*}$. Then ξ^* satisfies

$$p\int_0^{\xi^*} xdB(x) = c\mu. \tag{55}$$

When $Q_2 = Q_2^*$, the retailer's profit is

$$\Pi(Q_1, Q_2^*) = (p-c)Q_1 - H[Q_1 - N\overline{F}(p) - N\overline{F}(p + \frac{h}{1-\alpha^{\frac{1}{\gamma}}})] + p\overline{B}(\xi^*)[2N\overline{F}(p) - Q_1], \tag{56}$$

and Eqn. (52) becomes

$$\begin{aligned}
\alpha &= \overline{B}(\xi^*) + \int_0^{\xi^*} \frac{x[2N\overline{F}(p)-Q_1]/\xi^* + Q_1 - N\overline{F}(p) - N\overline{F}(p+\frac{h}{1-\alpha^{\frac{1}{\gamma}}})}{N\overline{F}(p) - N\overline{F}(p+\frac{h}{1-\alpha^{\frac{1}{\gamma}}})} dB(x) \\
&= \overline{B}(\xi^*) + B(\xi^*)\frac{Q_1 - N\overline{F}(p) - N\overline{F}(p+\frac{h}{1-\alpha^{\frac{1}{\gamma}}})}{N\overline{F}(p) - N\overline{F}(p+\frac{h}{1-\alpha^{\frac{1}{\gamma}}})} + \int_0^{\xi^*} \frac{x}{\xi^*} dB(x) \frac{2N\overline{F}(p)-Q_1}{N\overline{F}(p) - N\overline{F}(p+\frac{h}{1-\alpha^{\frac{1}{\gamma}}})} \\
&= 1 - [B(\xi^*) - \int_0^{\xi^*} \frac{x}{\xi^*} dB(x)] \frac{2N\overline{F}(p)-Q_1}{N\overline{F}(p) - N\overline{F}(p+\frac{h}{1-\alpha^{\frac{1}{\gamma}}})}.
\end{aligned} \tag{57}$$

Substituting Eqn. (57) into (56) and eliminating Q_1, we have

$$\begin{aligned}
\Pi(Q_1, Q_2^*) &= 2(p-c)N\overline{F}(p) - [N\overline{F}(p) - N\overline{F}(p+\frac{h}{1-\alpha^{\frac{1}{\gamma}}})] \times \\
&\quad \{(1-\alpha)[\frac{B(\xi^*)p - c}{B(\xi^*) - \int_0^{\xi^*} \frac{x}{\xi^*} dB(x)} - \frac{H}{B(\xi^*) - \int_0^{\xi^*} \frac{x}{\xi^*} dB(x)}] + H\}.
\end{aligned} \tag{58}$$

Note that $\int_0^{\xi^*} xdB(x) = c\mu/p$. Then if we let $\widehat{\beta} = 1 - B(\xi^*) + \int_0^{\xi^*} \frac{x}{\xi^*} dB(x)$, $\widehat{p} = p + \frac{(\mu-\xi^*)c}{(1-\widehat{\beta})\xi^*}$ and $\widehat{F}(x) = F(x - \frac{(\mu-\xi^*)c}{(1-\widehat{\beta})\xi^*})$, we can find that that the problem here is equivalent to Problem (12)-(13) in Section 5. Similar results in Section 5 can be generated to general distribution of yield proportion.

8. Conclusion

In this paper, we develop a model to study consumer panic buying under supply disruption and investigate how the retailer should respond to panic buying through inventory policy and fixed quota policy.

First, we develop a model to characterize consumer panic buying. We identify a threshold policy for the consumers: If a consumer's valuation of the product is above the threshold, the consumer will stockpile products for future consumption, while if his/her valuation is below the threshold, he will buy regular amount. Moveover, we find that the consumers are more likely to stockpile when the price of the product or the consumer holding cost is lower, or when the consumers are more risk averse, or less certain to obtain the product in Period 2.

Second, by an rational expectation equilibrium analysis, we charaterize the retailer's inventory policy facing consumer panic buying. We find that, it is not necessary for the retailer to carry inventory to hedge the risk of supply disruption when supply reliability is high, while it is optimal for the retailer to carry some inventory to hedge the risk of supply disruption when supply reliability is low. In particular, under the case that supply reliability is low, the retailer should carry more inventory to cover all demands in Period 2 if the consumers' desirability for the product is low; and the retailer should carry inventory to only cover partial demands in Period 2 if the consumers' desirability for the product is high.

Third, we investigate the cost of ignoring consumer behaviors. Interestingly, we find that the cost of ignoring consumer behaviors is not significant when supply reliability is very high or low, however, the cost is the most significant when supply reliability is intermediate, which is opposite to our intuition that the cost of ignoring consumer behaviors should be more significant when supply reliability is lower.

Fourth, we study impact of limited capacity and the effectiveness of the fixed quota policy when the retailer has limited capacity. We show that the fixed quota policy doesn't always benefit the retailer. It is shown that when the retailer's capacity is below a threshold, it should implement fixed quota policy, and if its capacity exceeds the threshold, it is better for the retailer not to implement fixed quota policy. We also show that the fixed quota policy is more attractive to the retailer when consumers' desirability for the product is higher, the consumers' holding cost is lower, the retailer's holding cost is lower, or the consumers' degree of risk aversion is higher. Interestingly, our numerical analysis demonstrates that attractiveness of the fixed quota policy increases in supply reliability and selling price when supply reliability is low, and it decreases in supply reliability or selling price when supply reliability is sufficiently high.

Finally, we provide two extensions to the basic model: i) responsive pricing (i.e., the retailer can change the price when supply disruption occurs), where we derive the equilibrium responsive price and identify conditions under which responsive pricing can strictly increase retailer's profit; ii) random yield model (i.e, the retailer get a random proportion of what it orders in Period 2), where we show that the managerial insights derived based on disruption model can be generalized into random yield model.

Acknowledgments

Appendix

Proof for Proposition 1.

Proof. We only show the proof that $T(p,h,\alpha_c)$ is increasing in p, since the proofs for h and α_c are similar. Since $U(x)$ is a increasing concave function, it can be verified that $U(v-p-h)/U(v-p)$ is increasing in v and decreasing in p. For any $p_1 < p_2$, we have

$$\frac{U(T(p_1,h,\alpha_c)-p_2-h)}{U(T(p_1,h,\alpha_c)-p_2)} < \frac{U(T(p_1,h,\alpha_c)-p_1-h)}{U(T(p_1,h,\alpha_c)-p_1)} = \alpha_c = \frac{U(T(p_2,h,\alpha_c)-p_2-h)}{U(T(p_2,h,\alpha_c)-p_2)},$$

which indicates that $T(p_1,h,\alpha_c) < T(p_2,h,\alpha_c)$ by that $U(v-p-h)/U(v-p)$ is increasing in v. ■

Proof for Proposition 2.

Proof. Recall the definition of T in Eqn. (5). Since it is easy to verify that both $U(v-p-h)/U(v-p)$ and $W(v-p-h)/W(v-p)$ are increasing in v, then one only needs to prove that

$$\frac{U(v-p-h)}{U(v-p)} \le \frac{W(v-p-h)}{W(v-p)} \tag{59}$$

for any $v \ge p+h$. Clearly, to prove Inequality (59), one only needs to prove that, for any x and y, it holds that

$$\frac{U(x-y)}{W(x-y)} \le \frac{U(x)}{W(x)} \tag{60}$$

Let $k(x) = U(x)/W(x)$. Then we have

$$k^{'}(x) = \frac{U^{'}(x)W(x) - W^{'}(x)U(x)}{[W(x)]^2} = \frac{U^{'}(x)[\varphi(U(x)) - \varphi^{'}(U(x))U(x)]}{[W(x)]^2}.$$

Since φ is a increasing concave function with $\varphi(0) = 0$, then

$$\varphi(U(x)) + \varphi^{'}(U(x))[0 - U(x)] \ge \varphi(0) = 0,$$

which indicates that $k(x)$ is a increasing function. Thus, Inequality (60) is true. ■

Proof for Proposition 3.

Proof. The proof is the same as Case (a) in the proof of Theorem 2.

Proof for Proposition 4

Proof. Part (i). We only provide a proof with respect to γ, since the others are obvious. Let

$$Y(\beta,\gamma) = \gamma\beta^{1-\frac{1}{\gamma}} + (1-\gamma)\beta - 1.$$

Differentiating $Y(\beta,\gamma)$ with respect to γ, we have

$$\frac{\partial Y}{\partial \gamma}(\beta,\gamma) = \beta^{1-\frac{1}{\gamma}}(1 + \frac{1}{\gamma}ln\beta) - \beta.$$

Since $\frac{\partial Y}{\partial \gamma}(1,\gamma) = 0$ and

$$\frac{\partial^2 Y}{\partial \gamma \partial \beta}(\beta,\gamma) = (1 + \frac{1}{\gamma}(1 - \frac{1}{\gamma})ln\beta)\beta^{-\frac{1}{\gamma}} - 1 > 0,$$

then for any $0 \leq \beta \leq 1$, we have $\frac{\partial Y}{\partial \gamma}(\beta,\gamma) \leq 0$, which implies that $\frac{\partial X}{\partial \gamma}(\beta,\gamma,p,H) \leq 0$, where $X(\beta,\gamma,p,H)$ is defined in (25). For any $0 < \gamma_1 < \gamma_2 \leq 1$, we have

$$X(\beta_t(\gamma_1,p,H),\gamma_2,p,H) \leq X(\beta_t(\gamma_1,p,H),\gamma_1,p,H) = 0,$$

which, together with the fact that $X(\beta,\gamma,p,H)$ is decreasing with respect to β, implies that $\beta_t(\gamma_1,p,H) \geq \beta_t(\gamma_2,p,H)$.

Part (ii). It can be easily verified that Π_2' in (23) is decreasing in β and H. Thus, by the concavity of Π_2 with respect to T, we have T^o is decreasing in β and H. To prove T^o increasing with respect to p, we denote $\widetilde{T} = T - p$. Then we have

$$g(\widetilde{T}) \equiv \Pi_2'(T) = -\frac{N}{\overline{v}}\{H - (p - \frac{H}{1-\beta})[(1 - \frac{h}{\widetilde{T}})^{\gamma-1}(1 - \frac{(1-\gamma)h}{\widetilde{T}}) - 1]\}. \tag{61}$$

It can be easily verified that $g(\widetilde{T})$ is decreasing in T and increasing in p. Hence, $\widetilde{T}^o$ such that $g(\widetilde{T}^o) = 0$ is increasing in p, which indicates that $T^o = \widetilde{T}^o + p$ is also increasing with respect to p. This completes the proof.

Proof for Theorem 4

Proof. We only provide a proof of Part (a) since that of Part (b) is similar. If $(\beta,\overline{v}) \in \Omega_2^n$, both Q^* and Q_I^* equal to $2N\overline{F}(p)$. If $(\beta,\overline{v}) \in \{(\beta,\overline{v}) \in \Omega_1^n | \overline{v} < p + \frac{h}{1-\beta}\}$, then $Q^* = Q_I^* = N\overline{F}(p)$. For other cases, $Q^* \neq Q_I^*$. If $Q^* \neq Q_I^*$ and $\beta \in (0, 1 - \frac{H}{p})$, $|Q^* - Q_I^*| = Q_I^* - Q^* = N[\overline{F}(p) - \overline{F}(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}})]$, which is increasing with respect to β. If $Q^* \neq Q_I^*$ and $\beta \in (1 - \frac{H}{p}, 1)$, then $|Q^* - Q_I^*| = Q^* - Q_I^* = N\overline{F}(p + \frac{h}{1-\beta^{\frac{1}{\gamma}}})$, which is decreasing with respect to β. ■

Proof for Theorem 5

Proof. Note that the retailer will implement fixed quota policy if and only if $\Pi_f^* \geq \Pi^*$. If $\beta > 1 - H/p$, then $\Pi^* - \Pi_f^* = pN\overline{F}(p)[(1-\beta\theta)\delta - 1]$. If $\beta \leq 1 - H/p$, then $\Pi^* - \Pi_f^* = N\overline{F}(p)\{[\beta p(1-\theta) + H]\delta - (H + \beta p)\}$. By the equations above, one can easily come to the results. ■

Proof for Proposition 5

Proof. From Eqn. (34), we know that δ_t is increasing in θ. Thus, δ_t is increasing in $\overline{v}$ and decreasing in γ and h by Lemma 1. By Eqn. (34), we have that, when $\beta \leq 1 - H/p$, $\delta_t = 1 + \frac{\theta\beta p}{\beta p(1-\theta)+H}$, which is obviously decreasing in H. When $\beta > 1 - H/p$, δ_t is regardless of H. Hence, δ_t is nonincreasing with respect to H.

Proof for Theorem 6

Proof. Part (1) is obvious. We only prove Part (2).

If $v \in (p, p_r)$, then $\pi_r(1, p_{r,c}, \eta_c) = (1+\beta)(v-p)$, $\pi_r(2, p_{r,c}, \eta_c) = 2(v-p) - h$. Thus, this part is true since it is a special case of Theorem 1.

If $v \geq p_{r,c}$, then $\pi_r(1, p_{r,c}, \eta_c) = (1+\beta)(v-p) + (1-\beta)\eta_c(v - p_{r,c})$, $\pi_r(2, p_{r,c}, \eta_c) = 2(v-p) - h$. The consumer will buy 1 (or 2) units in Period 1 is equivalent to $\pi_r(1, p_{r,c}, \eta_c) > (or\ <)\pi_r(2, p_{r,c}, \eta_c)$. By simple calculation, one can have Part (3) is true. ■

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